4.6 Rational Functions & Models

Rational Functions

A rational function is a function of the form

$$f(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.

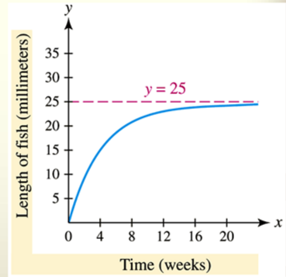
Examples of rational functions: $f(x) = \frac{1}{x}$

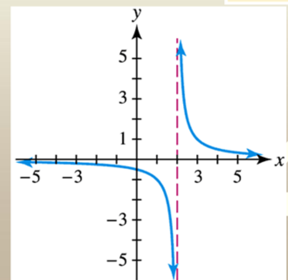
$$f(x) = \frac{x^2 - 3}{2x + 1}$$

$$g(x) = \frac{5x - 2}{x^2 - 3x - 4}$$

Asymptotes

- A rational function may have both vertical and horizontal asymptotes.
- A vertical asymptote occurs anywhere the denominator is zero and the numerator is not.





From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.207-8

Horizontal Asymptotes

- If the degree of the denominator is larger than the degree of the numerator, the horizontal asymptote is $y = 0$.
- If the degree of the numerator is larger than the degree of the denominator, there is no horizontal asymptote.
- If the degrees are the same, the horizontal asymptote is given by the ratio of the leading coefficients. That is, if

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0}{b_n x^n + b_{n-1} x^{n-1} + \ldots + b_1 x + b_0}$$

then $y = \frac{a_n}{b_n}$

Slant Asymptote

A slant asymptote (or oblique asymptote) may occur when the degree of the numerator is exactly one more than the degree of the denominator.

The equation of the slant asymptote is the quotient of performing long division.

To Graph a Rational Function

1. Find the vertical asymptotes by setting the denominator equal to zero.
2. Find the horizontal asymptote by comparing the degrees of the numerator & denominator.
3. Find the *y*-intercept by setting $x = 0$.
4. Find the *x*-intercept by setting the numerator equal to zero.
5. Sketch the graph (plot points to help, if necessary).

Examples

- Sketch $y = \dfrac{2x+3}{x+1}$
- Sketch $y = \dfrac{x^2-4}{x^2-x-6}$
- Sketch $y = \dfrac{4x^2+4x+1}{2x+1}$