

3.5 Transformations of Graphs

0011



Basic Graphs

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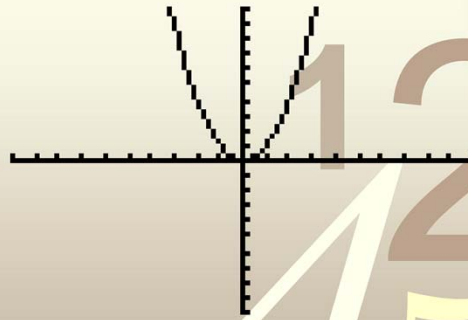
$$y = x^2$$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

No asymptotes

Passes through $(0, 0)$,
 $(1, 1)$, $(2, 4)$, $(-2, 4)$ and
so on.



Basic Graphs

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$$y = x^3$$

Domain: $(-\infty, \infty)$

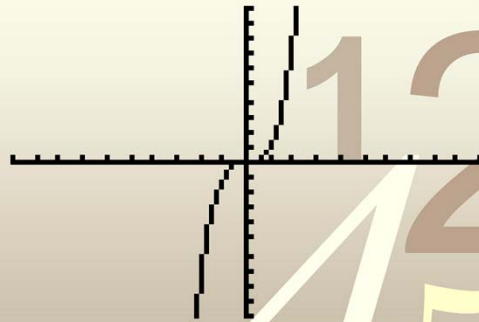
Range: $(-\infty, \infty)$

No asymptotes

Passes through $(0, 0)$,

$(1, 1)$, $(2, 8)$, $(-2, -8)$

and so on.



Basic Graphs

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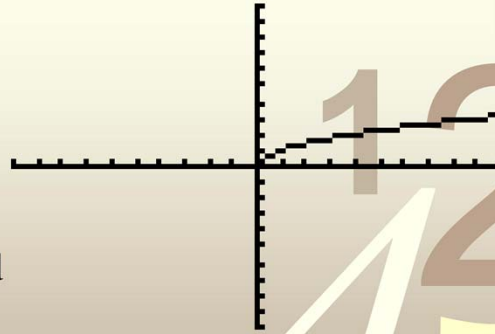
$$y = \sqrt{x}$$

Domain: $[0, \infty)$

Range: $[0, \infty)$

No asymptotes

Passes through $(0, 0)$,
 $(1, 1)$, $(4, 2)$, $(9, 3)$ and
so on.



Basic Graphs

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$$y = \sqrt[3]{x}$$

Domain: $(-\infty, \infty)$

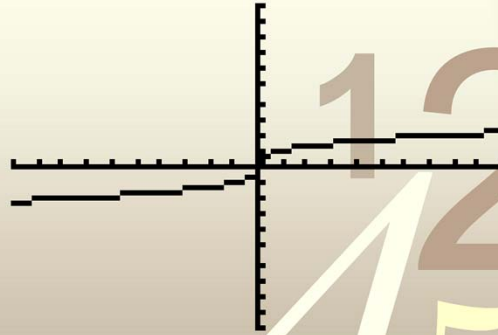
Range: $(-\infty, \infty)$

No asymptotes

Passes through $(0, 0)$,

$(1, 1)$, $(8, 2)$, $(-8, -2)$

and so on.



1
2
4
5

Basic Graphs

$$y = \frac{1}{x}$$

Domain: $(-\infty, 0) \cup (0, \infty)$

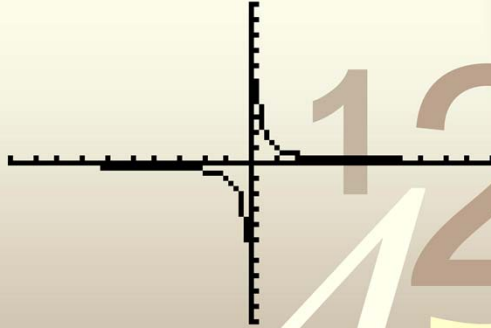
Range: $(-\infty, 0) \cup (0, \infty)$

Asymptotes

Horizontal $y = 0$

Vertical $x = 0$

Passes through $(1, 1)$,
 $(2, \frac{1}{2})$, $(-\frac{1}{2}, -2)$ and so
on.



Basic Graphs

$$y = \frac{1}{x^2}$$

Domain: $(-\infty, 0) \cup (0, \infty)$

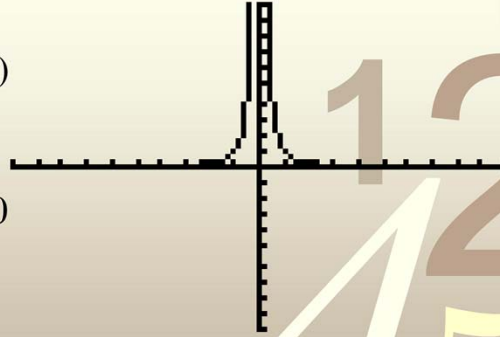
Range: $(0, \infty)$

Asymptotes

Horizontal $y = 0$

Vertical $x = 0$

Passes through $(1, 1)$,
 $(2, \frac{1}{4})$, $(-\frac{1}{2}, 4)$ and so
on.



Basic Graphs

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$$y = |x|$$

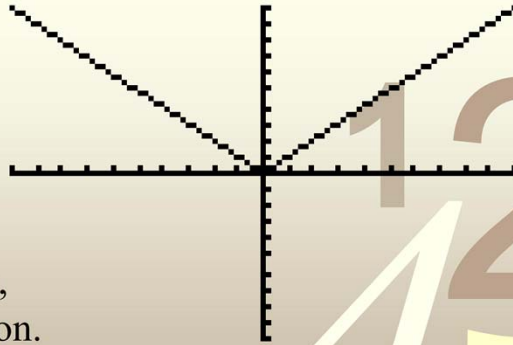
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Asymptotes

None

Passes through $(1, 1)$,
 $(2, 2)$, $(-2, 2)$ and so on.



Translations

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A translation of one of the basic graphs retains the same size and shape but has been shifted to a different location in the plane.



Vertical translations

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Compared with the graph of $y = f(x)$

- the graph of $y = f(x) + k$
is shifted upward k units
- the graph of $y = f(x) - k$
is shifted downward k
units

Assuming $k > 0$



Examples

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- Graph $f(x) = x^3 + 2$

- Graph $f(x) = \sqrt{x} - 4$



Horizontal translations

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Compared with the graph of $y = f(x)$

- the graph of $y = f(x + h)$
is shifted h units left
- the graph of $y = f(x - h)$
is shifted h units right

Assuming $h > 0$



Examples

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- Graph $f(x) = |x - 2|$
- Graph $f(x) = \frac{1}{x + 3}$
- Graph $f(x) = (x - 3)^2 + 2$

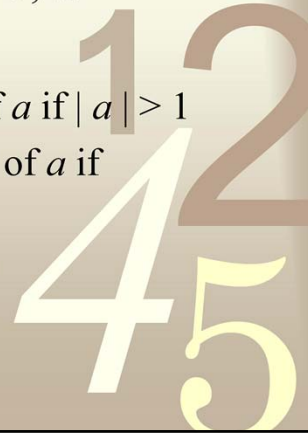


Vertical Stretching and Shrinking

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Compared with the graph of $y = f(x)$, the graph of $y = af(x)$, where $a \neq 0$, is

- expanded vertically by a factor of a if $|a| > 1$
- compressed vertically by a factor of a if $0 < |a| < 1$
- reflected about the x -axis if $a < 0$



Examples

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- Graph $f(x) = 2|x|$

- Graph $f(x) = -\frac{1}{2}\sqrt{x}$

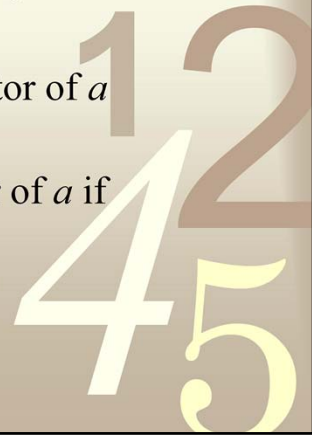


Horizontal Stretching and Shrinking

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Compared with the graph of $y = f(x)$, the graph of $y = f(ax)$, where $a \neq 0$, is

- compressed horizontally by a factor of a if $|a| > 1$
- expanded horizontally by a factor of a if $0 < |a| < 1$
- reflected about the y -axis if $a < 0$

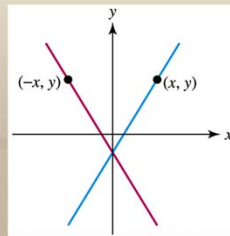
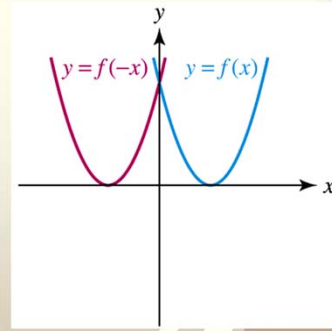
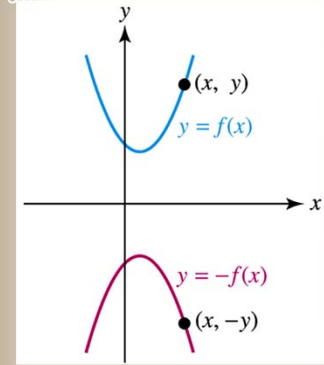


Reflections

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1. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ across the x -axis.
2. The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ across the y -axis.

Reflection of Graphs Across the x - and y -axes



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Examples

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- Graph $f(x) = \sqrt{-x}$
- Problem 34 on page 236 of your textbook

