3.1 Quadratic Functions & Models

Quadratic function

A quadratic function is a function of the form

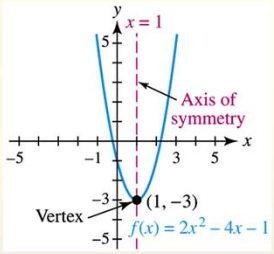
$$f(x) = ax^2 + bx + c$$

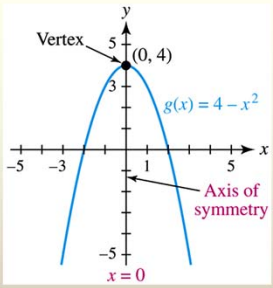
where a, b, and c are constants, and a is not equal to zero.

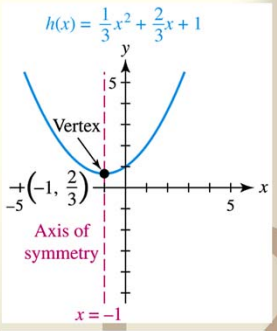
Features of the Graph of a Quadratic

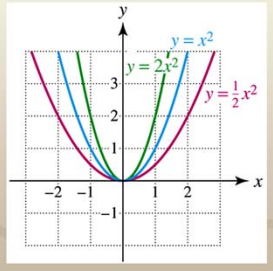
- The graph of $ax^2 + bx + c = y$ is a parabola
- The vertex is the highest or lowest point on the graph
- The axis of symmetry passes through the vertex

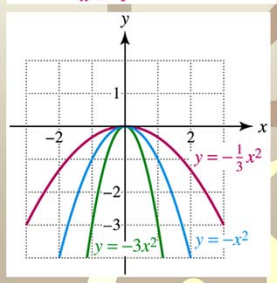
Examples of different parabolas











From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.173.

Graph of $y = ax^2 + bx + c$

The coefficient of x^2, a, determines the width of the graph.

- If $|a| < 1$, the graph is more wide than the graph of $y = x^2$
- If $|a| > 1$, the graph is more narrow than the graph of $y = x^2$

- If $a > 0$, the parabola opens up
- If $a < 0$, the parabola opens down

Vertex form

The vertex form of a quadratic equation is

$$y = a(x-h)^2 + k$$

where the vertex of the graph is

$$(h,k)$$

Example

Sketch the graph of

$$f(x) = (x+1)^2 - 2$$

Steps for completing the square

$$y = ax^2 + bx + c$$

1. Make sure the coefficient of x^2 is +1. If it is not, factor a out of the x terms only.
2. Square half the coefficient of x and add to the x terms and subtract this quantity from c (remember to multiply by a, if necessary).
3. Write the result as a perfect square and simplify the constant terms.

Examples

- Sketch the graph of $f(x) = x^2 + 10x + 7$
- Sketch the graph of $f(x) = 3x^2 + 6x + 2$
- Sketch the graph of $f(x) = -x^2 + 2x + 1$
- Sketch the graph of $f(x) = -\frac{1}{2}x^2 + x + 1$