

1.5 Functions and Their Rates of Change

0011

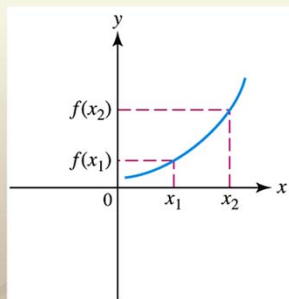


Increasing or Decreasing Functions

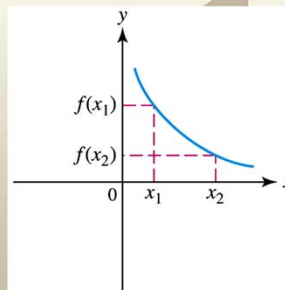
INCREASING AND DECREASING FUNCTIONS

001 Suppose that a function f is defined over an interval I on the number line. If x_1 and x_2 are in I ,

- (a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$;
- (b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$.



When $x_1 < x_2$, $f(x_1) < f(x_2)$,
 f is increasing.



When $x_1 < x_2$, $f(x_1) > f(x_2)$,
 f is decreasing.

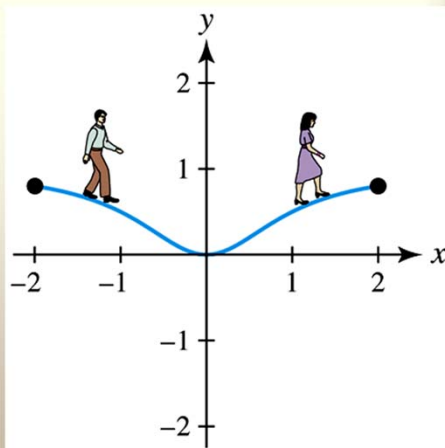
From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.243

Increasing or Decreasing Functions

0011

A graph that is increasing **RISES** as you read from left to right.

A graph that is decreasing **FALLS** as you read from left to right.



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.243

Interval Notation


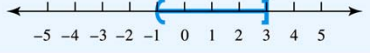
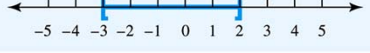
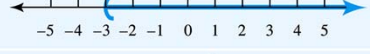
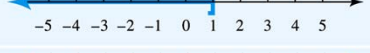
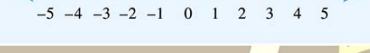
001

| Inequality | Interval | Inequality | Interval |
|-----------------------|---------------------------------|----------------------------------|----------|
| $x < a$ | $(-\infty, a)$ | $a < x < b$ | (a, b) |
| $x \leq a$ | $(-\infty, a]$ | $a \leq x < b$ | $[a, b)$ |
| $x > b$ | (b, ∞) | $a < x \leq b$ | $(a, b]$ |
| $x \geq b$ | $[b, \infty)$ | $a \leq x \leq b$ | $[a, b]$ |
| $x < a$ $x > b$ or | $(-\infty, a) \cup (b, \infty)$ | The union symbol represents "or" | |

Interval Notation

0011

TABLE 2.12 Interval Notation

| Inequality | Interval Notation | Graph |
|--|--|--|
| $-2 < x < 2$ | $(-2, 2)$ open interval |  |
| $-1 < x \leq 3$ | $(-1, 3]$ half-open interval |  |
| $-3 \leq x \leq 2$ | $[-3, 2]$ closed interval |  |
| $x > -3$ | $(-3, \infty)$ infinite interval |  |
| $x \leq 1$ | $(-\infty, 1]$ infinite interval |  |
| $-\infty < x < \infty$ (entire number line) | $(-\infty, \infty)$ infinite interval |  |

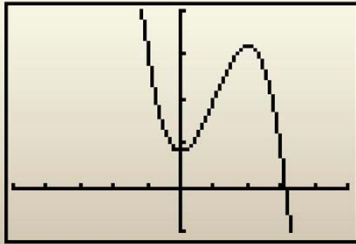
From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.125.

Examples

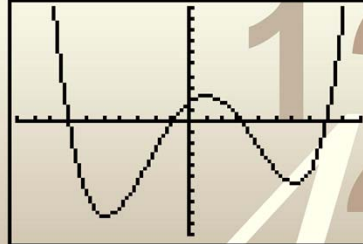
0011

Identify where the graph is increasing and decreasing

a.



b.



Average Rate of Change

0011

The average rate of change of f from x_1 to x_2 is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



Examples

0011

Find the average rate of change of the function over the specified interval.

a. $f(x) = 5x - 3$ **from** $x = -1$ **to** $x = 3$

b. $g(x) = 3 - 2x^2$ **from** $x = 2$ **to** $x = 7$

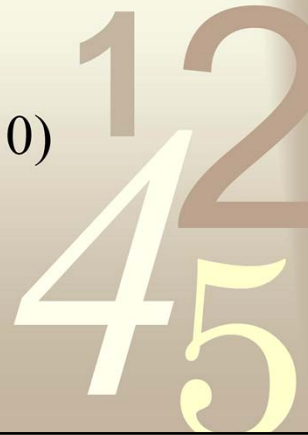
c. $H(x) = 3x^2 - 2x + 4$ **from** $x = a$ **to** $x = a + h$

Difference Quotient

0011

The formula for the average rate of change of a function can be rewritten as

$$\frac{f(x+h) - f(x)}{h} \quad (h \neq 0)$$



Examples

0011

Find the difference quotient for

a. $f(x) = -5x + 7$

b. $g(x) = -2x^2 - 8$

