9.7 Determinants

DETERMINANT OF A 2 × 2 MATRIX

The **determinant** of

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is a real number defined by

$$\det A = ad - cb.$$

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.825

Examples

Compute the determinant of the matrix A:

1. $A = \begin{bmatrix} 3 & 1 \\ 7 & -2 \end{bmatrix}$

2. $A = \begin{bmatrix} -4 & 10 \\ 2 & -5 \end{bmatrix}$

MINORS AND COFACTORS

The **minor**, denoted by M_{ij}, for element a_{ij} in the square matrix A is the real number computed by performing the following steps.

STEP 1: Delete the ith row and jth column from the matrix A.

STEP 2: M_{ij} is equal to the determinant of the resulting matrix.

The **cofactor**, denoted A_{ij}, for a_{ij} is defined by $A_{ij} = (-1)^{i+j} M_{ij}$.

Example: Find the minor M_{11} and the cofactor A_{11} for the matrix

$$A = \begin{bmatrix} -8 & 0 & 4 \\ 4 & -6 & 7 \\ 2 & -3 & 5 \end{bmatrix}$$

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.826

DETERMINANT OF A MATRIX USING THE METHOD OF COFACTORS

Multiply each element in any row or column of the matrix by its cofactor. The sum of the products is equal to the determinant.

Example: Find det A for

$$A = \begin{bmatrix} -8 & 0 & 4 \\ 4 & -6 & 7 \\ 2 & -3 & 5 \end{bmatrix}$$

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.827

CRAMER'S RULE FOR LINEAR SYSTEMS IN TWO VARIABLES

The solution to the linear system

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

is given by $x = \frac{E}{D}$ and $y = \frac{F}{D}$, where

$$E = \det\begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}, \quad F = \det\begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}, \quad \text{and} \quad D = \det\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \neq 0.$$

Example: Use Cramer's rule to solve the system

$$\begin{cases} x + y + 2z = 1 \\ -x - 2y - 3z = -2 \\ y - 3z = 5 \end{cases}$$

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.829