9.1 Functions and Systems Equations in Two Variables

Solving systems of two equations by substitution:

1. Solve one of the equations for one of the variables in terms of the other.
2. Substitute this expression into the 2nd equation, which will yield an equation in one variable.
3. Solve the new equation.
4. Use the result of step 1 to find the other variable.

Examples

Solve the system of linear equations:

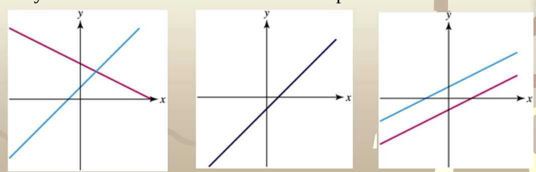
1. $\begin{cases} 7x - 2y = 5 \\ x + 9y = 10 \end{cases}$

2. $\begin{cases} \frac{1}{2}x - \frac{3}{4}y = \frac{1}{2} \\ \frac{1}{5}x - \frac{3}{10}y = \frac{1}{5} \end{cases}$

3. $\begin{cases} 0.6x - 0.2y = 2 \\ -1.2x + 0.4y = 3 \end{cases}$

Not every system of equations has a unique solution. There are three possibilities:

1. The graphs may be the same line. We say this is a dependent system.
2. The graphs may be parallel but distinct lines. We say this is an inconsistent system.
3. The graphs may intersect in one and only one point. The system is said to be consistent and independent.



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.755.

Solving a system of two equations by elimination

1. Choose one of the variables to eliminate. Multiply each equation by a suitable factor so that the coefficients of that variable are opposite.
2. Add the two new equations termwise.
3. Solve the resulting equation for the remaining variable.
4. Substitute the value found in step 3 into either of the original equations and solve for the other variable.

This is also called the method of linear combinations.

Examples

Solve the system of linear equations:

1. $\begin{cases} 4x+3y=8 \\ -2x+6y=1 \end{cases}$

2. $\begin{cases} -\frac{1}{3}x+\frac{1}{6}y=-1 \\ 2x-y=6 \end{cases}$

3. $\begin{cases} 5x-2y=7 \\ 10x-4y=6 \end{cases}$

Examples

Solve the system of equations:

1. $\begin{cases} x^2 + y^2 = 9 \\ x + y = 3 \end{cases}$

2. $\begin{cases} x^2 + y^2 = 4 \\ 2x^2 + y = -3 \end{cases}$