5.4 Logarithmic Functions & Models

Definition

The base b logarithm of x, written $\log_b x$ ($b > 0$ and $b \neq 1$), is the exponent to which b must be raised in order to yield x.

$y = \log_b x$ is equivalent to $b^y = x$

Examples

$2^3 = 8$ so $\log_2 8 = 3$

$3^2 = 9$ so $\log_3 9 = 2$

$5^3 = 125$ so $\log_5 125 = 3$

Examples

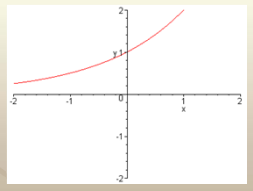
$\log_2 32 = 5$ because $2^5 = 32$

$\log_{10} 0.01 = -2$ because $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

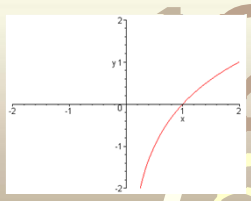
$\log_9 3 = \frac{1}{2}$ because $9^{1/2} = \sqrt{9} = 3$

$f(x) = \log_a x$ is the inverse function of $f(x) = a^x$

- Obtain the graph of the inverse by reflecting across $y = x$



$f(x) = a^x$



$f(x) = \log_a x$

Graph properties

$f(x) = a^x$	$f(x) = \log_a x$
• Domain $(-\infty, \infty)$	• Domain $(0, \infty)$
• Range $(0, \infty)$	• Range $(-\infty, \infty)$
• H. A. $y = 0$	• V. A. $x = 0$
• $(0,1)$	• $(1,0)$

Example

- Graph $y = \log_2 x$
- Graph $y = \log_2 (x + 5)$
- Graph $y = \log_2 (x) + 5$
- Graph $y = -3\log_2 (x + 1)$

Properties

- $\log_b b = 1$
- $\log_b 1 = 0$
- $\log_b b^x = x$
- $b^{\log_b x} = x$

Examples

- $\log_2 1 = 0$
- $\log_2 2 = 1$
- $\log_2 2^3 = 3$
- $2^{\log_2 5} = 5$

Common logarithm

The common logarithm is base 10. We often write it omitting the base:

$$\log x = \log_{10} x$$

Natural logarithm

The natural logarithm is base e. We write it as:

$$\ln x = \log_e x$$

Examples

- Solve $4^x = 11$
- Solve $2(10^x) = 66$
- Solve $\log_4 x = 3.7$
- Solve $16 - 4\ln 3x = 2$