# 5.3 Exponential Functions & Models

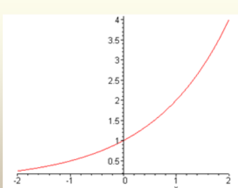
## Form

An exponential function is a function of the form

$$f(x) = b^x$$

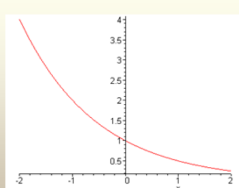
where $b > 0$ and $b \neq 1$.

## Shape of $y = b^x$



$b > 1$

increasing



$0 < b < 1$

decreasing

## Properties of the graph

- Domain is $(-\infty, \infty)$
- Range is $(0, \infty)$
- Horizontal Asymptote is $y = 0$
- $y$-intercept is $(0, 1)$

## Example

- Graph $y = 2^x$
- Graph $y = 2^{-x}$
- Graph $y = 2^x+1$
- Graph $y = 2^{x+1}$

## Compound Interest

The amount accumulated in an account bearing interest compounded $n$ times annually is

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

where $P$ = principal invested

$r$ = interest rate (as a decimal)

$t$ = time in years

## Example

Suppose $5000 is invested at an interest rate of 8%. Find the amount in the account after ten years if the interest is compounded

a. annually

b. semiannually

c. daily

## Definition of *e*

*e* is an irrational number (like π)

$$e \approx 2.718281828\,45$$

The natural exponential function is

$$f(x) = e^x$$

## Continuously Compounded Interest

The amount accumulated in an account bearing interest compounded continuously is

$$A(t) = Pe^{rt}$$

where $P$ = principal invested

$r$ = interest rate (as a decimal)

$t$ = time in years

## Example

$5000 is invested at an interest rate of 8%, compounded continuously. How much is in the account after 10 years?

## Exponential Growth and Decay

Exponential growth of a population is given by the formula

$$P(t) = P_0 e^{kt}$$

where

$P_0$ = initial size of the population

$k$ = relative rate of growth (positive) or decay (negative)

$t$ = time

## Example

The population of Phoenix, Arizona, in 2000 was 1.3 million and growing continuously at a 3% rate.

a. Assuming this trend continues, estimate the population of Phoenix in 2010.

b. Determine graphically or numerically when this population might reach 2 million.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.416, problem 98.