4.8 Power Functions & Radical Equations



PROPERTIES OF RATIONAL EXPONENTS

Let m and n be positive integers with $\frac{m}{n}$ in lowest terms and $n \ge 2$. Let r and p be rational numbers. Assume that b is a nonzero real number and that each expression is a real number.

Property

Example

1.
$$b^{m/n} = (b^m)^{1/n} = (b^{1/n})^m$$
 $4^{3/2} = (4^3)^{1/2} = (4^{1/2})^3 = 2^3 = 8$

2.
$$b^{m/n} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$$
 $8^{2/3} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

3.
$$(b^r)^p = b^{rp}$$

$$(2^{3/2})^4 = 2^6 = 64$$

4.
$$b^{-r} = \frac{1}{b^r}$$

$$5. b'b'' = b'$$

$$3^{5/2} \cdot 3^{3/2} = 3^{(5/2) + (3/2)} = 3^4 = 81$$

6.
$$\frac{b^r}{l^p} = b^{r-p}$$

$$\frac{5^{5/4}}{5^{3/4}} = 5^{(5/4) - (3/4)} = 5^{1/2}$$

From Precalculus with Modeling and Visualization 3rd ed. by Rockswold, 2006, p.335

Examples

• Solve
$$x^{\frac{2}{3}} = 16$$

• Solve
$$2n^{-2} - n^{-1} = 3$$

• Solve
$$x^{\frac{2}{3}} + 9x^{\frac{1}{3}} + 14 = 0$$

• Solve
$$x - 5 = \sqrt{5x - 1}$$

• Solve
$$\sqrt{2x-4} + 2 = \sqrt{3x+4}$$

