4.5 The Fundamental Theorem of Algebra

Number of Zeros of Polynomials

Fundamental Theorem of Algebra

The polynomial $f(x)$ of degree $n \geq 1$ has at least one complex zero.

Number of Zeros Theorem

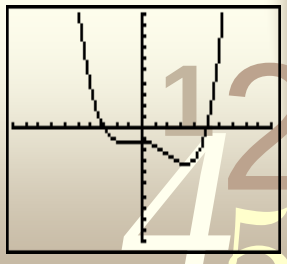
A polynomial of degree n has at most n distinct zeros.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.296-7

Example

The graph provided is a degree 4 polynomial.

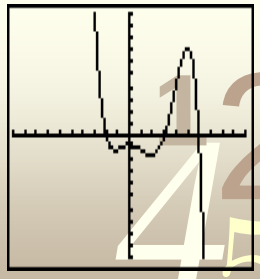
1. Identify whether the leading coefficient is positive or negative.
2. How many real and how many imaginary zeros does the polynomial have?



Example

The graph provided is a degree 5 polynomial.

1. Identify whether the leading coefficient is positive or negative.
2. How many real and how many imaginary zeros does the polynomial have?



Complex Zeros of Polynomials

Conjugate Zeros Theorem

If a polynomial $f(x)$ has only real coefficients and if $a + bi$ is a zero of $f(x)$, then the conjugate $a - bi$ is also a zero of $f(x)$.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.299

Examples

- Find the equation of a a degree 3 polynomial with leading coefficient – ¾ and zeros – 3*i* and 2/5.
- Given that 2i is one zero, find all the zeros of $f(x) = x^4 + 2x^3 + 8x^2 + 8x + 16$
- Find all the zeros of $f(x) = x^3 + 2x^2 + 16x + 32$