4.4 Real Zeros of Polynomial Functions

FACTOR THEOREM

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Example: Use the factor theorem to decide if $x - \frac{1}{2}$ is a factor of $f(x) = 2x^4 - 11x^3 + 9x^2 + 14x$

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.279

COMPLETE FACTORED FORM

Suppose a polynomial

$$f(x) = a_n x^n + \cdots + a_2 x^2 + a_1 x + a_0$$

has n real zeros $c_1, c_2, c_3, \ldots, c_n$, where distinct zeros are listed as many times as their multiplicities. Then $f(x)$ can be written in **complete factored form** as

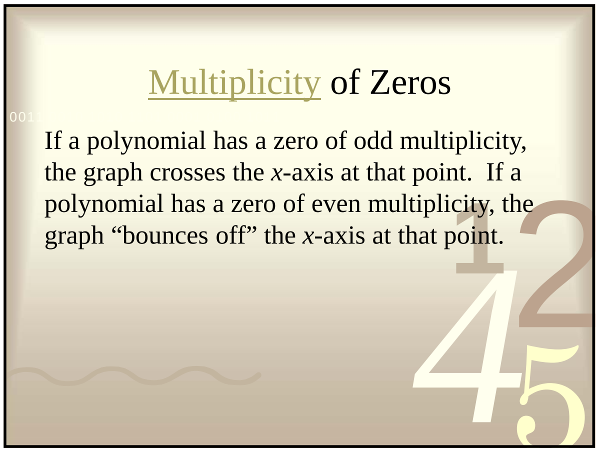
$$f(x) = a_n(x - c_1)(x - c_2)(x - c_3) \cdots (x - c_n).$$

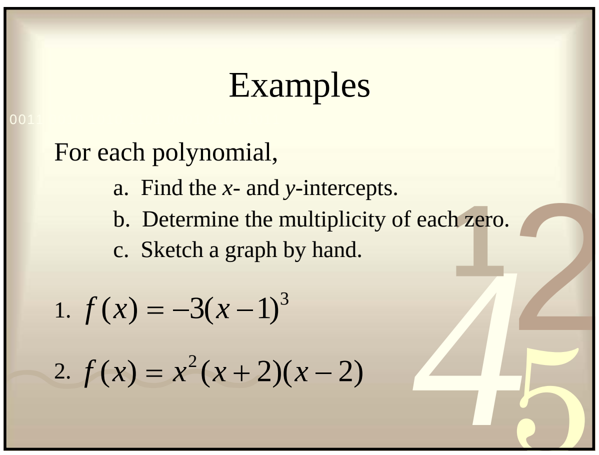
Example: Write the complete factored form of

$$f(x) = 2x^3 + x^2 - 11x - 10$$

given that -2 is a zero.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.280


Multiplicity of Zeros
If a polynomial has a zero of odd multiplicity, the graph crosses the x-axis at that point. If a polynomial has a zero of even multiplicity, the graph "bounces off" the x-axis at that point.


Examples
For each polynomial,
a. Find the x- and y-intercepts.
b. Determine the multiplicity of each zero.
c. Sketch a graph by hand.
1. $f(x) = -3(x-1)^3$
2. $f(x) = x^2(x+2)(x-2)$