4.1 Nonlinear Functions & Their Graphs

Definition

A polynomial function has the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$$

where $a_0, a_1, a_2, \ldots, a_n$ are constants

Examples: $x^5 + 5x^2 - 3$

$x^3 - 2x^2 + 1$

$7x^2 - 2$

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Degree of a Polynomial

In a polynomial containing only one variable, the greatest exponent that appears on the variable is called the <u>degree</u> of the polynomial

Examples:

$x^8 - 3x^2 + 1$	is degree 8
$-2x^4 - x^6$	is degree 6
5	is degee 0
x	is degree 1

Absolute and Local Extrema

ABSOLUTE AND LOCAL EXTREMA

Let c be in the domain of f.

$f(c)$ is an **absolute (global) maximum** if $f(c) \geq f(x)$ *for all* x in the domain of f.

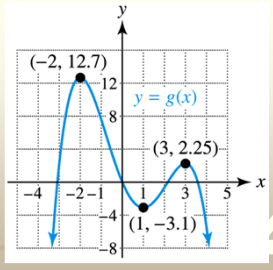
$f(c)$ is an **absolute (global) minimum** if $f(c) \leq f(x)$ *for all* x in the domain of f.

$f(c)$ is a **local (relative) maximum** if $f(c) \geq f(x)$ when x is *near* c.

$f(c)$ is a **local (relative) minimum** if $f(c) \leq f(x)$ when x is *near* c.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.245

Absolute and Local Extrema

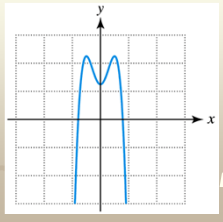


From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.245

Even Function

EVEN FUNCTION

A function f is an **even function** if $f(-x) = f(x)$ for every x in its domain. The graph of an even function is symmetric with respect to the y-axis.

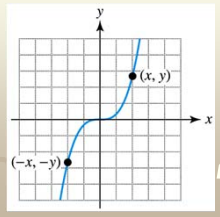


From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.247

Odd Function

ODD FUNCTION

A function f is an **odd function** if $f(-x) = -f(x)$ for every x in its domain. The graph of an odd function is symmetric with respect to the origin.



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.248

Examples

Determine if the function is even, odd, or neither.

a. $f(x) = x^4 - 5x^2 + 2$

b. $g(x) = x^3 - 2x^2 + 5$

c. $h(x) = x^3 - 3x + 1$

d. $k(x) = \frac{1}{x} + 5$

e. graphically