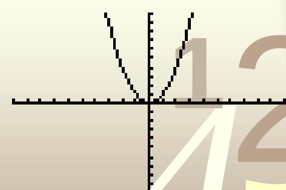
3.5 Transformations of Graphs

Basic Graphs

$y = x^2$

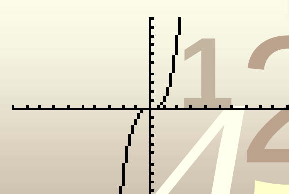
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
No asymptotes
Passes through (0, 0), (1, 1), (2, 4), (-2, 4) and so on.



Basic Graphs

$y = x^3$

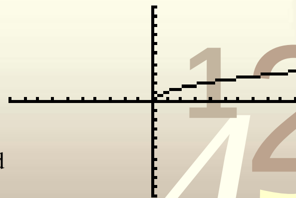
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
No asymptotes
Passes through (0, 0), (1, 1), (2, 8), (-2, -8) and so on.



Basic Graphs

$$y = \sqrt{x}$$

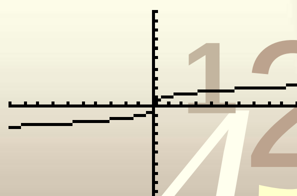
Domain: [0, ∞)
Range: [0, ∞)
No asymptotes
Passes through (0, 0), (1, 1), (4, 2), (9, 3) and so on.



Basic Graphs

$$y = \sqrt[3]{x}$$

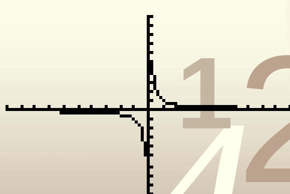
Domain: (-∞, ∞)
Range: (-∞, ∞)
No asymptotes
Passes through (0, 0), (1, 1), (8, 2), (-8, -2) and so on.



Basic Graphs

$$y = \frac{1}{x}$$

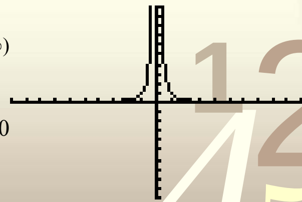
Domain: $(-\infty,0)\cup(0,\infty)$
Range: $(-\infty,0)\cup(0,\infty)$
Asymptotes
  Horizontal $y = 0$
  Vertical $x = 0$
Passes through (1, 1), (2, ½), (-½, -2) and so on.



Basic Graphs

$$y = \frac{1}{x^2}$$

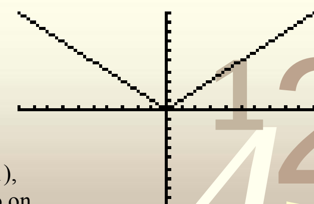
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(0, \infty)$
Asymptotes
Horizontal $y = 0$
Vertical $x = 0$
Passes through (1, 1), (2, ¼), (-½, 4) and so on.



Basic Graphs

$$y = |x|$$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Asymptotes
None
Passes through (1, 1), (2, 2), (-2, 2) and so on.



Translations

A translation of one of the basic graphs retains the same size and shape but has been shifted to a different location in the plane.

Vertical translations

Compared with the graph of $y = f(x)$

– the graph of $y = f(x) + k$ is shifted upward k units

– the graph of $y = f(x) - k$ is shifted downward k units

Assuming $k > 0$

Examples

- Graph $f(x) = x^3 + 2$
- Graph $f(x) = \sqrt{x} - 4$

Horizontal translations

Compared with the graph of $y = f(x)$

– the graph of $y = f(x + h)$ is shifted h units left

– the graph of $y = f(x - h)$ is shifted h units right

Assuming $h > 0$

Examples

- Graph $f(x) = |x - 2|$
- Graph $f(x) = \dfrac{1}{x+3}$
- Graph $f(x) = (x-3)^2 + 2$

Vertical Stretching and Shrinking

Compared with the graph of $y = f(x)$, the graph of $y = af(x)$, where $a \neq 0$, is

- expanded vertically by a factor of a if $|a| > 1$
- compressed vertically by a factor of a if $0 < |a| < 1$
- reflected about the x-axis if $a < 0$

Examples

- Graph $f(x) = 2|x|$
- Graph $f(x) = -\dfrac{1}{2}\sqrt{x}$

Horizontal Stretching and Shrinking

Compared with the graph of $y = f(x)$, the graph of $y = f(ax)$, where $a \neq 0$, is

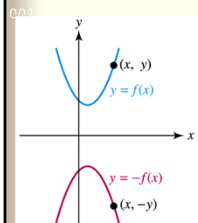
- compressed horizontally by a factor of a if $|a| > 1$
- expanded horizontally by a factor of a if $0 < |a| < 1$
- reflected about the y-axis if $a < 0$

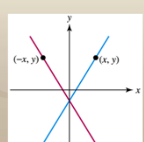
Reflections

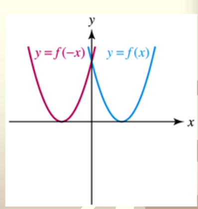
1. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ across the x-axis.

2. The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ across the y-axis.

Reflection of Graphs Across the x- and y-axes







Examples

- Graph $f(x) = \sqrt{-x}$

- Problem 34 on page 236 of your textbook