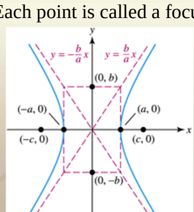
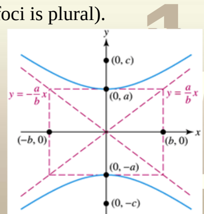
10.3 Hyperbolas

Hyperbola

- A hyperbola is formed by all points in the plane, the difference of whose distances from two fixed points is a constant.
- Each point is called a focus (foci is plural).





From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.871

STANDARD EQUATIONS FOR HYPERBOLAS CENTERED AT (0, 0)

The hyperbola with center at the origin, *horizontal* transverse axis, and equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

has asymptotes $y = \pm\frac{b}{a}x$, vertices $(\pm a, 0)$, and foci $(\pm c, 0)$, where $c^2 = a^2 + b^2$.

The hyperbola with center at the origin, *vertical* transverse axis, and equation

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

has asymptotes $y = \pm\frac{a}{b}x$, vertices $(0, \pm a)$, and foci $(0, \pm c)$, where $c^2 = a^2 + b^2$.

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Examples

Graph the hyperbola, including the asymptotes. Give the coordinates of the foci.

1. $49y^2 - 25x^2 = 1225$

2. $4x^2 - 4y^2 = 100$

STANDARD EQUATIONS FOR HYPERBOLAS CENTERED AT (h, k)

A hyperbola with center (h, k), and either a horizontal or vertical transverse axis, satisfies one of the following equations, where $c^2 = a^2 + b^2$.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Transverse axes: horizontal
Vertices: $(h \pm a, k)$; foci: $(h \pm c, k)$
Asymptotes: $y = \pm \frac{b}{a}(x - h) + k$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Transverse axes: vertical
Vertices: $(h, k \pm a)$; foci: $(h, k \pm c)$
Asymptotes: $y = \pm \frac{a}{b}(x - h) + k$

Shifted Hyperbolas

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.875

Examples

1. Write the equation
   $$4x^2 + 16x - 9y^2 + 18y = 29$$
   in the standard form for a hyperbola. Graph the hyperbola and identify the center and the vertices.
2. Find the equation of a hyperbola with vertices (2±1, 1) and foci (2±3, 1).