

10.2 Ellipses

Ellipse

- An ellipse is formed by all points in the plane, the sum of whose distances from two fixed points is a constant.
- Each point is called a focus (foci is plural).

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.856

STANDARD EQUATIONS FOR ELLIPSES CENTERED AT (0, 0)

The ellipse with center at the origin, *horizontal* major axis, and equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b > 0)$$

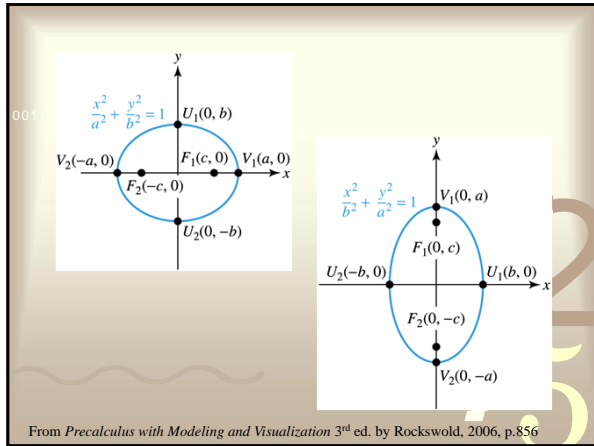
has vertices $(\pm a, 0)$, endpoints of the minor axis $(0, \pm b)$, and foci $(\pm c, 0)$, where $c^2 = a^2 - b^2$ and $c \geq 0$.

The ellipse with center at the origin, *vertical* major axis, and equation

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (a > b > 0)$$

has vertices $(0, \pm a)$, endpoints of the minor axis $(\pm b, 0)$, and foci $(0, \pm c)$, where $c^2 = a^2 - b^2$ and $c \geq 0$.

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Examples

Graph the ellipse. Label the foci and the endpoints of each axis.

1. $x^2 + 4y^2 = 400$
2. $5x^2 + 4y^2 = 20$

STANDARD EQUATIONS FOR ELLIPSES CENTERED AT (h, k)

An ellipse with center (h, k) , and either a horizontal or vertical major axis, satisfies one of the following equations, where $a > b > 0$ and $c^2 = a^2 - b^2$ with $c \geq 0$.

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	Major axis: horizontal; foci: $(h \pm c, k)$ Vertices: $(h \pm a, k)$
$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	Major axis: vertical; foci: $(h, k \pm c)$ Vertices: $(h, k \pm a)$

Shifted Ellipses

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.849

Examples

1. Write the equation

$$9x^2 - 36x + 16y^2 - 64y - 44 = 0$$

in the standard form for an ellipse. Graph the ellipse and identify the center and the vertices.

2. Find the equation of an ellipse with vertices $(-1, \pm 3)$ and foci $(-1, \pm 1)$.

Circles

- A circle is a type of an ellipse.
- The standard form for a circle is with radius r and center (h, k) is

$$(x-h)^2 + (y-k)^2 = r^2$$

Example

- Find the standard equation of a circle with center $(-1, -3)$ passing through the point $(3, 0)$. Graph the circle.
