

1.5 Functions and Their Rates of Change

Increasing or Decreasing Functions

INCREASING AND DECREASING FUNCTIONS

Suppose that a function f is defined over an interval I on the number line. If x_1 and x_2 are in I ,

(a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$;
 (b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$.

When $x_1 < x_2$, $f(x_1) < f(x_2)$,
 f is increasing.

When $x_1 < x_2$, $f(x_1) > f(x_2)$,
 f is decreasing.

From Precalculus with Modeling and Visualization 3rd ed. by Rockswold, 2006, p.243

Increasing or Decreasing Functions

A graph that is increasing **RISES** as you read from left to right.

A graph that is decreasing **FALLS** as you read from left to right.

From Precalculus with Modeling and Visualization 3rd ed. by Rockswold, 2006, p.243

Interval Notation

Inequality	Interval	Inequality	Interval
$x < a$	$(-\infty, a)$	$a < x < b$	(a, b)
$x \leq a$	$(-\infty, a]$	$a \leq x < b$	$[a, b)$
$x > b$	(b, ∞)	$a < x \leq b$	$(a, b]$
$x \geq b$	$[b, \infty)$	$a \leq x \leq b$	$[a, b]$
$x < a$ or $x > b$	$(-\infty, a) \cup (b, \infty)$	The union symbol represents "or"	

Interval Notation

TABLE 2.12 Interval Notation

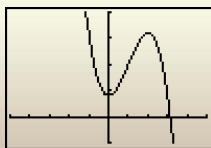
Inequality	Interval Notation	Graph
$-2 < x < 2$	$(-2, 2)$ open interval	
$-1 < x \leq 3$	$(-1, 3]$ half-open interval	
$-3 \leq x \leq 2$	$[-3, 2]$ closed interval	
$x > -3$	$(-3, \infty)$ infinite interval	
$x \leq 1$	$(-\infty, 1]$ infinite interval	
$-\infty < x < \infty$ (entire number line)	$(-\infty, \infty)$ infinite interval	

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.125.

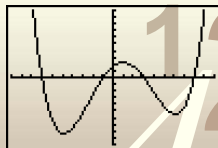
Examples

Identify where the graph is increasing and decreasing

a.



b.



Average Rate of Change

The average rate of change of f from x_1 to x_2 is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

12
45

Examples

Find the average rate of change of the function over the specified interval.

- a. $f(x) = 5x - 3$ from $x = -1$ to $x = 3$
- b. $g(x) = 3 - 2x^2$ from $x = 2$ to $x = 7$
- c. $H(x) = 3x^2 - 2x + 4$ from $x = a$ to $x = a + h$

12
45

Difference Quotient

The formula for the average rate of change of a function can be rewritten as

$$\frac{f(x+h) - f(x)}{h} \quad (h \neq 0)$$

12
45

Examples

001

Find the difference quotient for

a. $f(x) = -5x + 7$

b. $g(x) = -2x^2 - 8$

12
45
