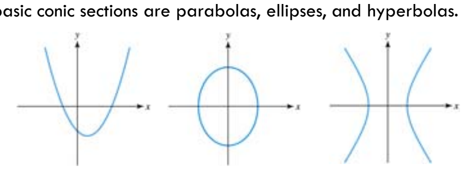
OBJECTIVE 7

Conic Sections

CONIC SECTIONS

Conic sections are formed when a plane intersects a cone in different ways.

The basic conic sections are parabolas, ellipses, and hyperbolas.

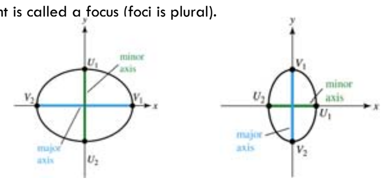


From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.845

ELLIPSE

An ellipse is formed by all points in the plane, the sum of whose distances from two fixed points is a constant.

Each point is called a focus (foci is plural).



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.856

STANDARD EQUATIONS FOR ELLIPSES CENTERED AT (0, 0)

The ellipse with center at the origin, *horizontal* major axis, and equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (a > b > 0)$$

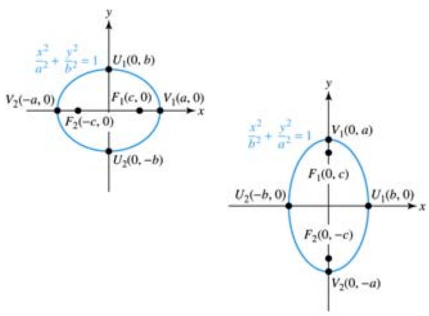
has vertices $(\pm a, 0)$, endpoints of the minor axis $(0, \pm b)$, and foci $(\pm c, 0)$, where $c^2 = a^2 - b^2$ and $c \geq 0$.

The ellipse with center at the origin, *vertical* major axis, and equation

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \qquad (a > b > 0)$$

has vertices $(0, \pm a)$, endpoints of the minor axis $(\pm b, 0)$, and foci $(0, \pm c)$, where $c^2 = a^2 - b^2$ and $c \geq 0$.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.856



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.856

ELLIPSES

Graph the ellipse. Label the foci and the endpoints of each axis.

1. $x^2 + 4y^2 = 400$
2. $5x^2 + 4y^2 = 20$

STANDARD EQUATIONS FOR ELLIPSES CENTERED AT (h, k)

An ellipse with center (h, k), and either a horizontal or vertical major axis, satisfies one of the following equations, where $a > b > 0$ and $c^2 = a^2 - b^2$ with $c \geq 0$.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Major axis: horizontal; foci: $(h \pm c, k)$
Vertices: $(h \pm a, k)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Major axis: vertical; foci: $(h, k \pm c)$
Vertices: $(h, k \pm a)$

Shifted Ellipses

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.849

ELLIPSES

1. Write the equation $9x^2 - 36x + 16y^2 - 64y - 44 = 0$ in the standard form for an ellipse. Graph the ellipse and identify the center and the vertices.
2. Find the equation of an ellipse with vertices $(-1, \pm 3)$ and foci $(-1, \pm 1)$.
3. Write the equation $x^2 + 4y^2 - 8y + 4x - 8 = 0$ in the standard form for an ellipse. Graph the ellipse and identify the center and the vertices.
4. Find the equation of an ellipse with vertices $(\pm 6, 0)$ and foci $(\pm 4, 0)$.

CIRCLES

A circle is a type of an ellipse.

The standard form for a circle is with radius *r* and center (*h*, *k*) is

$$(x-h)^2 + (y-k)^2 = r^2$$

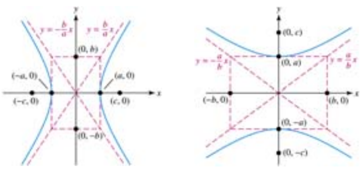
CIRCLES

1. Find the standard equation of a circle with center $(-1, -3)$ passing through the point (3, 0). Graph the circle.
2. Find the standard equation of a circle whose diameter has endpoints (4, 9) and (-2, 1).
3. Identify the center and radius of the circle $x^2 + y^2 - 12x - 10y + 52 = 0$, then sketch its graph.

HYPERBOLA

A hyperbola is formed by all points in the plane, the difference of whose distances from two fixed points is a constant.

Each point is called a focus (foci is plural).



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.871

STANDARD EQUATIONS FOR HYPERBOLAS CENTERED AT (0, 0)

The hyperbola with center at the origin, *horizontal* transverse axis, and equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

has asymptotes $y = \pm\frac{b}{a}x$, vertices $(\pm a, 0)$, and foci $(\pm c, 0)$, where $c^2 = a^2 + b^2$.

The hyperbola with center at the origin, *vertical* transverse axis, and equation

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

has asymptotes $y = \pm\frac{a}{b}x$, vertices $(0, \pm a)$, and foci $(0, \pm c)$, where $c^2 = a^2 + b^2$.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.871

HYPERBOLAS

Graph the hyperbola, including the asymptotes. Give the coordinates of the foci.

1. $49y^2 - 25x^2 = 1225$
2. $4x^2 - 4y^2 = 100$

STANDARD EQUATIONS FOR HYPERBOLAS CENTERED AT (h, k)

A hyperbola with center (h, k), and either a horizontal or vertical transverse axis, satisfies one of the following equations, where $c^2 = a^2 + b^2$.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Transverse axes: horizontal
Vertices: $(h \pm a, k)$; foci: $(h \pm c, k)$
Asymptotes: $y = \pm \frac{b}{a}(x - h) + k$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Transverse axes: vertical
Vertices: $(h, k \pm a)$; foci: $(h, k \pm c)$
Asymptotes: $y = \pm \frac{a}{b}(x - h) + k$

Shifted Hyperbolas

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.875

HYPERBOLAS

1. Write the equation $4x^2 + 16x - 9y^2 + 18y = 29$ in the standard form for a hyperbola. Graph the hyperbola and identify the center and the vertices.
2. Find the equation of a hyperbola with vertices $(2\pm1, 1)$ and foci $(2\pm3, 1)$.
3. Write the equation $8(x-4)^2 - 3(y-3)^2 = 24$ in the standard form for a hyperbola. Graph the hyperbola and identify the center and the vertices.
4. Find the equation of a hyperbola with vertices $(-4\pm2\sqrt{5}, 1)$ and asymptotes $y = \frac{1}{2}x + 3$ and $y = -\frac{1}{2}x - 1$

PARABOLA

A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line. The fixed point is called the **focus** and the fixed line is called the **directrix** of the parabola.

EQUATION OF A PARABOLA WITH VERTEX (0, 0)

Vertical Axis

The parabola with a focus at $(0, p)$ and directrix $y = -p$ has equation

$$x^2 = 4py.$$

The parabola opens upward if $p > 0$ and downward if $p < 0$.

Horizontal Axis

The parabola with a focus at $(p, 0)$ and directrix $x = -p$ has equation

$$y^2 = 4px.$$

The parabola opens to the right if $p > 0$ and to the left if $p < 0$.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.845-6

PARABOLAS

Graph the parabola. Label the vertex, focus, and directrix.

1. $y = -\frac{1}{8}x^2$
2. $3x = \frac{1}{2}y^2$

EQUATION OF A PARABOLA WITH VERTEX (*h*, *k*)

$(x - h)^2 = 4p(y - k)$	Vertical axis; vertex: (h, k) $p > 0$: opens upward; $p < 0$: opens downward Focus: $(h, k + p)$; directrix: $y = k - p$
$(y - k)^2 = 4p(x - h)$	Horizontal axis; vertex: (h, k) $p > 0$: opens to the right; $p < 0$: opens to the left Focus: $(h + p, k)$; directrix: $x = h - p$

Shifted Parabolas

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.849

PARABOLAS

1. Write the equation $y^2 + 8x - 8 = 4x$ in the standard form for a parabola. Graph the parabola and label the vertex, focus, and directrix.
2. Find the equation of a parabola with Focus (2, 1) and directrix $x = -1$.
3. Write the equation $x^2 - 14x - 24y + 1 = 0$ in the standard form for a parabola. Graph the parabola and label the vertex, focus, and directrix.
4. Find the equation of a parabola with Focus (0, 2) and directrix $y = -2$.

SYSTEMS OF EQUATIONS

Solve the system of equations:

1. $\begin{cases} x^2 + y^2 = 9 \\ x + y = 3 \end{cases}$
2. $\begin{cases} x^2 + y^2 = 4 \\ 2x^2 + y = -3 \end{cases}$
3. $\begin{cases} 5x^2 - 2y^2 = 75 \\ 2x^2 + 3y^2 = 125 \end{cases}$

INEQUALITIES

Graph the solution set to the inequality or system of inequalities:

1. $2x^2 - y < 1$
2. $\begin{cases} x^2 + y \le 4 \\ x^2 - y \le 3 \end{cases}$
3. $\begin{cases} x^2 + y^2 \le 25 \\ x + 2y \le 5 \end{cases}$