OBJECTIVE 2

Functions & Graphs

GRAPH

1. $y = 3x + 2$
2. $y = -2x - 4$
3. $y = \frac{5}{3}x + 1$
4. $y = 4x^2$
5. $y = -\frac{2}{3}x^3$
6. $2x - 3y = 18$
7. $y = 5$

GRAPH

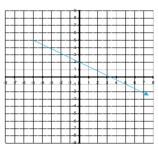
1. $y = -x + 4$
2. $y = 2x - 3$
3. $y = -\frac{1}{4}x + 3$
4. $y = -\frac{1}{4}x^2$
5. $y = 3x^3$
6. $4x + 3y = 2$
7. $x = 1$

DOMAIN AND RANGE

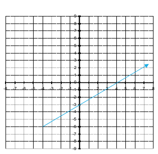
State the domain and range of each relation:

1. $\{(-2,4),(-3,-5),(-1,3),(4,,-5),(2,-3)\}$
2. $\{(-1,1),(0,4),(2,-5),(-3,4),(2,3)\}$

FIND THE EQUATION OF THE LINE SHOWN



FIND THE EQUATION OF THE LINE SHOWN



FIND THE EQUATION

1. Find the equation of the line with slope $\frac{2}{3}$ that passes through the point $(5, 1)$ in slope-intercept form.
2. Find the equation of the line between the points $(2, 3)$ and $(-1, 7)$ in slope-intercept form.
3. Find the equation of the vertical line that passes through $(-4, 1)$.
4. Find the equation of the line parallel to $2x - 3y = 8$ that passes through $(-1, 7)$ in slope-intercept form.
5. Find the equation of the line perpendicular to $5x + 2y = 4$ that passes through $(2, -8)$ in slope-intercept form.

FIND THE EQUATION

1. Find the equation of the line with slope $-\frac{1}{3}$ that passes through the point $(3, 5)$ in slope-intercept form.
2. Find the equation of the line between the points $(-1, -8)$ and $(2, -14)$ in slope-intercept form.
3. Find the equation of the horizontal line that passes through $(2, 17)$.
4. Find the equation of the line parallel to $3x + 3y = 5$ that passes through $(2, 9)$ in slope-intercept form.
5. Find the equation of the line perpendicular to $-x + 2y = 7$ that passes through $(1, 1)$ in slope-intercept form.

GAS MILEAGE

When empty, a large dump-truck gets about 15 miles per gallon. It is estimated that for each 3 tons of cargo it hauls, gas mileage decreases by ¾ mile per gallon.

1. If 10 tons of cargo is being carried, what is the truck's mileage?
2. If the truck's mileage is down to 10 miles per gallon, how much weight is it carrying?

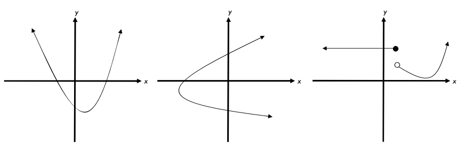
BASEBALL CARD VALUE

After purchasing an autographed baseball card for \$85, its value increases by \$1.50 per year.

1. What is the card's value 7 years after purchase?
2. How many years will it take for this card's value to reach \$100?

VERTICAL LINE TEST

If no vertical line intersects a graph in more than one place, then the graph represents a function.



EVALUATING A FUNCTION

For the given function, evaluate at each value

a. $f(x) = 2x^2 - 5x + 3$

- **i.** $f(0)$
- **ii.** $f(-4)$

b. $h(z) = \sqrt{z}$

- **i.** $h(0)$
- **ii.** $h(49)$
- **iii.** $h(-9)$

c. $g(y) = 3y + 2$

- **i.** $g(-1)$
- **ii.** $g(y+h)$
- **iii.** $\frac{g(y+h) - g(y)}{h}$

EVALUATING A FUNCTION

For the given function, evaluate at each value

a. $g(x) = -3x^2 + x - 7$

i. $g(-1)$

ii. $g(5y)$

iii. $\frac{g(x+h) - g(x)}{h}$

DOMAIN & RANGE

The domain consists of all values of the independent variable, *x*, allowed in the function.

The range consists of all values of the dependent variable, *f*(*x*) or *y*, that result as the independent variable takes on values across the domain.

RULES FOR FINDING THE DOMAIN OF AN ALGEBRAIC FUNCTION

1. If the function is a fraction, set the denominator equal to zero and solve for *x*. These are the values to EXCLUDE from the domain.
2. If the function is an EVEN radical, set what's under the radical greater than or equal to zero. Solve for *x* to obtain the domain.
3. The above rules can be combined.
4. If neither of the first two rules applies, the domain is all real numbers.

DOMAIN

Find the domain

1. $f(x) = 2x - 5, \quad -3 \le x \le 7$
2. $g(t) = \sqrt{t}$
3. $h(x) = \dfrac{1}{2x-5}$
4. $f(p) = \dfrac{2p+3}{\sqrt{p}}$

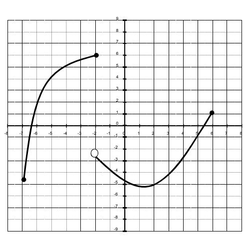
DOMAIN

Find the domain

1. $f(x) = \dfrac{2x-5}{x^2+4}$
2. $g(t) = \sqrt{3t-2}$
3. $h(x) = \dfrac{5x+1}{8x-1}$
4. $f(p) = \dfrac{\sqrt{x-4}}{x-18}$

DOMAIN & RANGE

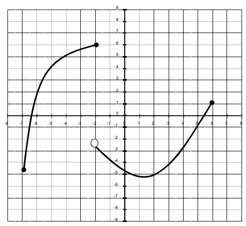
Find the domain and range



EVALUATING FUNCTIONS

Given the graph of a function, *f*, find

1. $f(-6) =$
2. $f(-2) =$
3. $f(x) = 5$

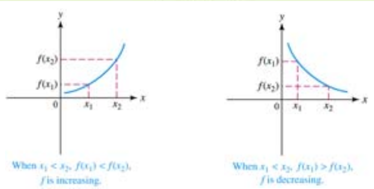


INCREASING OR DECREASING FUNCTIONS

INCREASING AND DECREASING FUNCTIONS

Suppose that a function f is defined over an interval I on the number line. If x_1 and x_2 are in I,

(a) f **increases** on I if, whenever $x_1 < x_2$, $f(x_1) < f(x_2)$;
(b) f **decreases** on I if, whenever $x_1 < x_2$, $f(x_1) > f(x_2)$.

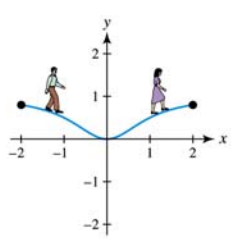


From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.243

INCREASING OR DECREASING FUNCTIONS

A graph that is increasing RISES as you read from left to right.

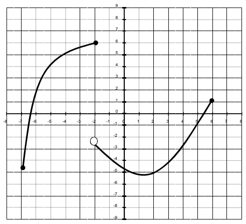
A graph that is decreasing FALLS as you read from left to right.



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.243

INCREASING & DECREASING

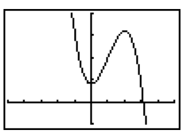
Determine where the function below is increasing and decreasing



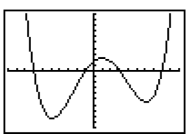
INCREASING & DECREASING

Identify where the graph is increasing and decreasing

a.

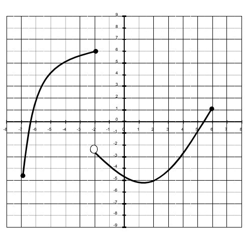


b.



INTERCEPTS

Find the *x*- and *y*-intercepts of the function shown



AVERAGE RATE OF CHANGE

The average rate of change of f from x_1 to x_2 is

$$\frac{f(x_2)-f(x_1)}{x_2-x_1}$$

EXAMPLES

Find the average rate of change of the function over the specified interval.

a. $f(x)=5x-3$ **from** $x=-1$ **to** $x=3$

b. $g(x)=3-2x^2$ **from** $x=2$ **to** $x=7$

c. $H(x)=3x^2-2x+4$ **from** $x=a$ **to** $x=a+h$

DIFFERENCE QUOTIENT

The formula for the average rate of change of a function can be rewritten as

$$\frac{f(x+h)-f(x)}{h} \quad (h\neq 0)$$

EXAMPLES

Find the difference quotient for

a. $f(x) = -5x + 7$

b. $g(x) = -2x^2 - 8$

ABSOLUTE AND LOCAL EXTREMA

ABSOLUTE AND LOCAL EXTREMA

Let c be in the domain of f.

$f(c)$ is an **absolute (global) maximum** if $f(c) \geq f(x)$ *for all* x in the domain of f.

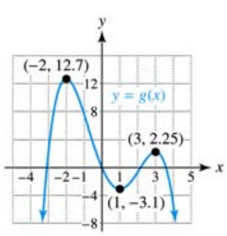
$f(c)$ is an **absolute (global) minimum** if $f(c) \leq f(x)$ *for all* x in the domain of f.

$f(c)$ is a **local (relative) maximum** if $f(c) \geq f(x)$ when x is *near* c.

$f(c)$ is a **local (relative) minimum** if $f(c) \leq f(x)$ when x is *near* c.

From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.245

ABSOLUTE AND LOCAL EXTREMA

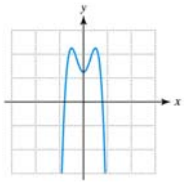


From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.245

EVEN FUNCTION

EVEN FUNCTION

A function f is an **even function** if $f(-x) = f(x)$ for every x in its domain. The graph of an even function is symmetric with respect to the y-axis.

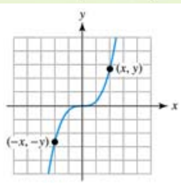


From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.247

ODD FUNCTION

ODD FUNCTION

A function f is an **odd function** if $f(-x) = -f(x)$ for every x in its domain. The graph of an odd function is symmetric with respect to the origin.



From *Precalculus with Modeling and Visualization* 3rd ed. by Rockswold, 2006, p.248

EXAMPLES

Determine if the function is even, odd, or neither.

a. $f(x) = x^4 - 5x^2 + 2$

b. $g(x) = x^3 - 2x^2 + 5$

c. $h(x) = x^3 - 3x + 1$

d. $k(x) = \frac{1}{x} + 5$

e. graphically

ABSOLUTE VALUE

Recall, the absolute value of *x*, | *x* |, is the distance of *x* from the origin (always positive). We can define the absolute value of *x* piecewise as

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

EXAMPLE

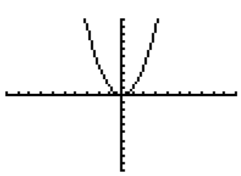
Rewrite as a piecewise defined function and sketch the graph:

$$g(x) = |1 - x|$$

BASIC GRAPHS

$$y = x^2$$

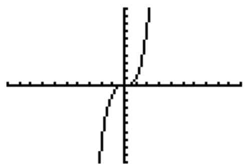
Domain: (-∞, ∞)
Range: [0, ∞)
No asymptotes
Passes through (0, 0), (1, 1), (2, 4), (-2, 4) and so on.



BASIC GRAPHS

$$y = x^3$$

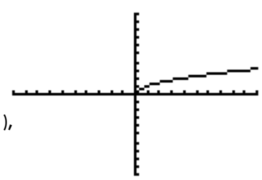
Domain: (-∞, ∞)
Range: (-∞, ∞)
No asymptotes
Passes through (0, 0), (1, 1), (2, 8), (-2, -8) and so on.



BASIC GRAPHS

$$y = \sqrt{x}$$

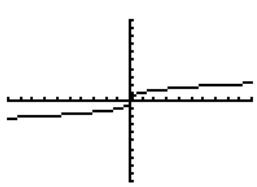
Domain: [0, ∞)
Range: [0, ∞)
No asymptotes
Passes through (0, 0), (1, 1), (4, 2), (9, 3) and so on.



BASIC GRAPHS

$$y = \sqrt[3]{x}$$

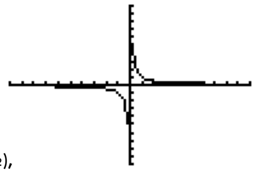
Domain: (-∞, ∞)
Range: (-∞, ∞)
No asymptotes
Passes through (0, 0), (1, 1), (8, 2), (-8, -2) and so on.



BASIC GRAPHS

$$y = \frac{1}{x}$$

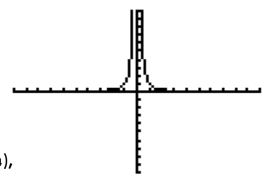
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Asymptotes
Horizontal $y = 0$
Vertical $x = 0$
Passes through (1, 1), (2, ½), (-½, -2) and so on.



BASIC GRAPHS

$$y = \frac{1}{x^2}$$

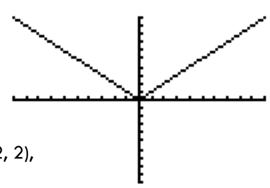
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(0, \infty)$
Asymptotes
Horizontal $y = 0$
Vertical $x = 0$
Passes through (1, 1), (2, ¼), (-½, 4) and so on.



BASIC GRAPHS

$$y = |x|$$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Asymptotes
None
Passes through (1, 1), (2, 2), (-2, 2) and so on.



TRANSLATIONS

A translation of one of the basic graphs retains the same size and shape but has been shifted to a different location in the plane.

VERTICAL TRANSLATIONS

Compared with the graph of $y = f(x)$,

- the graph of $y = f(x) + k$ is shifted upward k units
- the graph of $y = f(x) - k$ is shifted downward k units

Assuming $k > 0$

TRANSFORMATIONS

Graph

1. $f(x) = x^3 + 2$
2. $f(x) = \sqrt{x} - 4$

HORIZONTAL TRANSLATIONS

Compared with the graph of $y = f(x)$,

- the graph of $y = f(x + h)$ is shifted *h* units left
- the graph of $y = f(x - h)$ is shifted *h* units right

Assuming $h > 0$

TRANSFORMATIONS

Graph

1. $f(x) = |x - 2|$
2. $f(x) = \frac{1}{x+3}$
3. $f(x) = (x - 3)^2 + 2$

VERTICAL STRETCHING AND SHRINKING

Compared with the graph of $y = f(x)$, the graph of $y = af(x)$, where $a \neq 0$, is

- expanded vertically by a factor of *a* if $|a| > 1$
- compressed vertically by a factor of *a* if $0 < |a| < 1$
- reflected about the *x*-axis if $a < 0$

TRANSFORMATIONS

Graph

1. $f(x) = 2|x|$
2. $f(x) = -\frac{1}{2}\sqrt{x}$

HORIZONTAL STRETCHING AND SHRINKING

Compared with the graph of $y = f(x)$, the graph of $y = f(ax)$, where $a \neq 0$, is

- compressed horizontally by a factor of a if $|a| > 1$
- expanded horizontally by a factor of a if $0 < |a| < 1$
- reflected about the y-axis if $a < 0$

TRANSFORMATIONS

Graph

1. $f(x) = \sqrt{-x}$
2. $g(x) = 3\sqrt{-x+2} - 1$

PIECEWISE-DEFINED FUNCTIONS

Piecewise-defined functions are defined by different rules on different parts of their domain.

Help for sketching the graph is available on my website under the "General Handouts" link.

CONTINUITY

A function is continuous where you can draw the graph without lifting your pen from the paper.

A function is continuous where it has no holes, breaks, jumps, gaps, or asymptotes.

PIECEWISE-DEFINED FUNCTIONS

For the function

$$f(x)=\begin{cases}2x+1 & \text{if } -3\le x<0\\ x-1 & \text{if } 0\le x\le 3\end{cases}$$

a. Determine the domain.
b. Evaluate $f(-2)$, $f(0)$, and $f(3)$.
c. Graph f.
d. Is f continuous on its domain?

PIECEWISE-DEFINED FUNCTIONS

For the function

$$f(x)=\begin{cases}-3 & if\ x<-2\\ 4x+1 & if\ -2\le x\end{cases}$$

a. Determine the domain.
b. Evaluate f(-4), f(-2), and f(0).
c. Graph *f*.
d. Is *f* continuous on its domain?

OPERATIONS ON FUNCTIONS

For two functions, *f* and *g*, we define the new functions

$(f+g)(x)=f(x)+g(x)$ with domain D

$(f-g)(x)=f(x)-g(x)$ with domain D

$(fg)(x)=f(x)\cdot g(x)$ with domain D

$(f/g)(x)=f(x)\ /\ g(x)$ with domain D and

$g(x)\neq 0$

Where D is the domain that *f* and g have in common.

ALGEBRAIC OPERATIONS ON FUNCTIONS

For the functions given, find $f+g$, $f-g$, fg, and $f\ /\ g$.

A. $f(x)=\frac{1}{x}$ and $g(x)=x^3$

B. $f(x)=6-x$ and $g(x)=\sqrt{x-4}$

COMPOSITION OF FUNCTIONS

For the functions *f* and *g*, we define the composition function

$$(f \circ g)(x) = f(g(x))$$

The domain is all values of *x* in the domain of *g* for which *g*(*x*) is in the domain of *f*.

COMPOSITE FUNCTIONS

If $f(x) = \sqrt{4-x}$ and $g(x) = x^2$, find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

State the domain of each

COMPOSITE FUNCTIONS

If $f(x) = \frac{1}{3x}$ and $g(x) = \frac{2}{x-1}$, find

a. $(f \circ g)(x)$

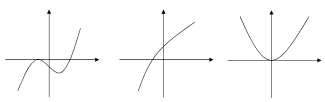
b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

State the domain of each

HORIZONTAL LINE TEST

If no horizontal line intersects the graph of a function more than once, then the function is a one-to-one function.



INVERSE OF A FUNCTION

If $f(x)$ is a function, its inverse, denoted $f^{-1}(x)$, “undoes” what f “does”.

That is, if $f(a) = b$ then $f^{-1}(b) = a$

Note: $f^{-1} \neq \frac{1}{f}$

RELATIONSHIP BETWEEN A FUNCTION AND ITS INVERSE

The domain of f is the range of f^{-1} and the range of f is the domain of f^{-1}.

If (a, b) is an ordered pair on the graph of f, then (b, a) is an ordered pair on the graph of f^{-1}.

The graph of f^{-1} is the graph of f reflected about the line $y = x$.

RELATIONSHIP BETWEEN A FUNCTION AND ITS INVERSE

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

TO FIND THE INVERSE OF A FUNCTION:

Example $f(x) = 5x$

Write the function as $y = f(x)$	$y = 5x$
Exchange x & y	$x = 5y$
Solve for y	$y = \frac{1}{5}x$
Write the solution using inverse notation	$f^{-1}(x) = \frac{1}{5}x$

INVERSE FUNCTIONS

For the function given, find $f^{-1}(x)$

1. $f(x) = 1 - \frac{1}{2}x^3$
2. $f(x) = \frac{x-1}{2}$
3. $f(x) = \frac{3x}{x-1}$
4. $f(x) = x^4 - 1$

INVERSE FUNCTIONS

For the function given, find $f^{-1}(x)$

1. $f(x) = 5x + 3$
2. $f(x) = \frac{5x+1}{2x-3}$
3. $f(x) = \sqrt{x} - 4$