

Piecewise-defined functions

To learn how to correctly graph piecewise-defined functions, you need colored pencils and graph paper. Obtain some before you continue. I will illustrate the procedure with an example.

Example 1 Graph $f(x) = \begin{cases} 2 & \text{if } x \geq -1, \\ -2x + 4 & \text{if } x < -1 \end{cases}$.

Solution 1 Take your yellow colored pencil and draw a vertical line along the x -axis at $x = -1$. Why at $x = -1$? Because that is the value in the domain where our piecewise-defined function changes rules. Think of $f(x)$ as being described by two functions: the top one (which has equation $y = 2$) and the bottom one (which has equation $y = -2x + 4$).

Now, take your blue colored pencil and underline the top equation in the formula for $f(x)$. Then draw the graph of that function (the horizontal line $y = 2$). Read the condition on $f(x)$ that corresponds to the piece you just drew. This is the graph of $f(x)$ when $x \geq -1$ so to the right of the yellow line (right because the inequality says x is **greater than or equal to**) write the word “keep” in your blue pencil above the graph.

Now, take your red colored pencil and underline the bottom equation in the formula for $f(x)$. Then draw the graph of that function (the line $y = -2x + 4$). This is a line with a slope of -2 and a y -intercept at $(0, 4)$. So start at the point $(0, 4)$ and plot points by moving down two and right one **or** up two and left one. Read the condition on $f(x)$ that corresponds to the piece you just drew. This is the graph of $f(x)$ when $x < -1$ so to the left of the yellow line (left because the inequality says x is **less than**) write the word “keep” in your red pencil above the graph.

Now, get your eraser. Erase the red and the blue that is not on the side of the yellow line that has the appropriate color of the word keep above it. Then erase both appearances of the word “keep”. Set your eraser down.

Remember that in order for $f(x)$ to be a function, we cannot have two different y -values for one x -value. But we do here: the red and the blue both touch the yellow line once. So, one of them must be an open circle and the other is the closed circle. To determine which one is closed, look at the equations you underlined in the formula for $f(x)$. The one with the equal sign is underlined in the color (blue) that will get a closed circle and the other color (red) will get an open circle.

Now, erase the yellow line. Trace over everything in blue and red with the color black (but be sure not to try to connect it! Remember to trace the pieces exactly). You're done. You can repeat this process for however many conditions your equation has (you might need several colors).