Finding the Domain of an Algebraic Function

To find the domain of an algebraic function, we must realize that there are two things that could give us difficulty: a fraction and an even root.

Fractions

A fraction cannot have a zero in the denominator because division by zero is an operation that is not defined. **Our only concern is eliminating the zeros of the denominator from the domain**. It's ok if the numerator is zero–we can divide into zero (if we do, we get zero) but we are not allowed to divide by zero.

Procedure: *Set the denominator of the fraction equal to zero and solve for x. Eliminate these values from the domain.*

Example 1 *Find the domain of the function* $f(x) = \frac{2-x}{x^2-4x-21}$.

Solution 1 *We only need to eliminate the zeros of the denominator.*

$$x^2 - 4x - 21 \neq 0$$

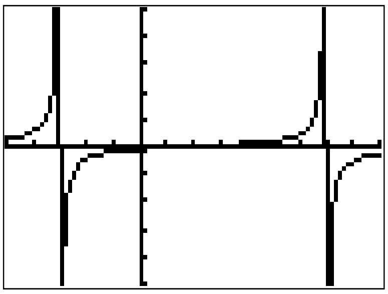
$$(x-7)(x+3) \neq 0$$

$$x - 7 \neq 0 \textit{ and } x + 3 \neq 0$$

$$x \neq 7 \textit{ and } x \neq -3$$

Then the domain is $(-\infty, -3) \bigcup (-3, 7) \bigcup (7, \infty)$.

On the graph below, we see vertical lines across the x-axis where $x = -3$ and $x = 7$.



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These vertical lines represent the asymptotes (they're not really there, your calculator is just filling in the gap left by the function). The function does not take on a value at these points because division by zero is undefined. Notice also that the numerator of the fraction is not simultaneously zero at these two points. If it were, we'd have a hole in the graph rather than an asymptote (but these values still would not be in the domain).

Even Radicals

If an even radical has a negative under the square root, it is an imaginary number. In most of the work we'll be doing, we are trying to graph equations along the real number line. Consequently, we need to avoid imaginary numbers. Note, however, that a negative under an odd root causes no problem.

Procedure: *Set whatever is underneath the radical greater than or equal to zero and solve this inequality. These are the values allowed in your domain.*

Example 2 *Find the domain of* $y = \sqrt{8 - x}$.

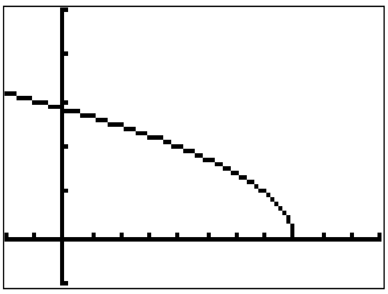
Solution 2 *We establish the inequality described by the procedure (note: this ensures that the quantity* $8 - x$ *will be nonnegative).*

$$8 - x \geq 0$$

$$8 \geq x$$

Thus, the domain is $(-\infty, 8]$.

On the graph below, we see the function takes on no values to the right of $x = 8$. This is because these values are not in the domain.



Combinations of the Two

You may also encounter combinations of the above two rules. Below are two examples.

Example 3 *Find the domain of* $f(x) = \frac{\sqrt{x+3}}{2-x}$.

Solution 3 *We analyze the two pieces separately. Because of the numerator we must require*

$$x + 3 \geq 0$$

$$x \geq -3$$

and the denominator requires that

$$2 - x \neq 0$$

$$2 \neq x$$

Combining the two restrictions $x \geq -3$ *and* $2 \neq x$*, we find the domain is* $[-3, 2) \bigcup (2, \infty)$.

Example 4 *Find the domain of* $g(x) = \frac{2-x}{\sqrt{x+3}}$.

Solution 4 *This time we will completely ignore the numerator. It will not affect the domain. Looking at the denominator, we set*

$$x + 3 \geq 0$$

because we have an even radical. But because we have a fraction, we cannot have a zero in the denominator; that is,

$$x + 3 \neq 0$$

This puts a further restriction on the inequality we developed. Instead of being a weak inequality ($\geq$) *it has to be a strict inequality* ($>$)*. This will always be the case:* ***If an even radical is the denominator of a fraction, instead of setting it greater than or equal to zero, set it strictly greater than zero****. So we have*

$$x + 3 > 0$$

$$x > -3$$

Thus, the domain is $(-3, \infty)$.

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