Completing the Square

We wish to complete the square for the expression $ax^2 + bx + c$.

Step 1: Make sure the coefficient of x^2 is 1. In this general case, it is not so we factor a outside a set of parentheses containing the terms involving x:

$$a(x^2 + \frac{b}{a}x) + c$$

Step 2: Right now, we are only interested in working inside the parentheses. Since $\frac{b}{a}$ is cumbersome to work with, we will just call this number b_1 so that our expression looks like

$$a(x^2 + b_1 x) + c$$

Take half of b_1, square it, and add it inside the parentheses. When you do this, you must also subtract it from the outside of the parentheses so that you don't change the value of the expression. But remember, the inside of the parentheses is being multiplied by a so what we subtract outside of the parentheses must also be multiplied by a:

$$a\left[x^2 + b_1 x + \left(\frac{b_1}{2}\right)^2\right] + c - a\left(\frac{b_1}{2}\right)^2$$

Step 3: The brackets above always factor to the perfect square

$$a\left[x + \frac{b_1}{2}\right]^2 + c - a\left(\frac{b_1}{2}\right)^2$$

We then combine the last two terms into one number to simplify.

Example 1 *Complete the square for* $x^2 + 3x - 3$.

Solution 1 *Here,* $a = 1$ *so we don't need to factor it outside a set of parentheses. Then* $b = b_1 = 3$ *so*

$$\frac{b}{2} = \frac{3}{2}$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

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Now we add and subtract this value from the expression:

$$\left[x^2 + 2x + \frac{9}{4}\right] - 3 - \frac{9}{4}$$

Rewrite the perfect square and simplify:

$$\left(x + \frac{3}{2}\right)^2 - \frac{12}{4} - \frac{9}{4}$$

$$\left(x + \frac{3}{2}\right)^2 - \frac{21}{4}$$

Example 2 *Complete the square for* $-2x^2 + 10x + 7$.

Solution 2 *Here,* $a = -2$ *so we factor it outside a set of parentheses:*

$$-2\left(x^2 - \frac{10}{2}x\right) + 7$$

$$-2(x^2 - 5x) + 7$$

Then $b_1 = -5$ *so*

$$\frac{b_1}{2} = \frac{-5}{2}$$

$$\left(\frac{b_1}{2}\right)^2 = \left(\frac{-5}{2}\right)^2 = \frac{25}{4}$$

Now we add and subtract this value from the expression remembering to multiply by a:

$$-2\left[x^2 - 5x + \frac{25}{4}\right] + 7 - (-2) \cdot \frac{25}{4}$$

Rewrite the perfect square and simplify:

$$-2\left(x - \frac{5}{2}\right)^2 + \frac{28}{4} + \frac{50}{4}$$

$$-2\left(x - \frac{5}{2}\right)^2 + \frac{78}{4}$$

$$-2\left(x - \frac{5}{2}\right)^2 + \frac{39}{2}$$