MAP 2302 Sample Problems, Exam 1

1. Give the order of the differential equation $\frac{d^2x}{dy^2} - 3x = \sin y$. State whether the equation is linear or nonlinear.
2. Use the Existence of a Unique Solution Theorem to determine if a unique solution exists for the initial value problem $y' = \sqrt{xy}$, $y(1) = 0$.
3. Obtain the direction field for $y' = 2x - y$.
4. Find the critical points and phase portrait of $\frac{dy}{dx} = y^2(4 - y^2)$. Sketch typical solution curves in the regions in the xy-plane determined by the graphs of the equilibrium solutions.
5. Solve $x\sqrt{1+y^2}\,dx = y\sqrt{1+x^2}\,dy$.
6. Solve $(1+x)\frac{dy}{dx} - xy = x + x^2$. Give the largest interval over which the general solution is defined.
7. Determine whether $(x^3 + y^3)\,dx + 3xy^2\,dy = 0$ is exact. If it is exact, solve it.
8. Solve the initial value problem $(x^2 + y^2 - 5)\,dx = (y + xy)\,dy$, $y(0) = 1$ by finding an appropriate integrating factor.
9. Solve $\frac{dy}{dx} = \frac{x+3y}{3x+y}$ by using an appropriate substitution
10. Solve $\frac{dy}{dx} - y = e^x y^2$ by using an appropriate substitution.
11. Solve $\frac{dy}{dx} = \sin(x + y)$ by using an appropriate substitution.
12. Initially 100 milligrams of a radioactive substance was present. After 6 hours the mass had decreased by 3%. If the rate of decay is proportional to the amount of substance present at time t, find the amount remaining after 24 hours.

Solutions:

1. 2nd order linear
2. A unique solution is not guaranteed by the theorem
3.
4. Critical points $y = 0, \pm 2$
5. $\sqrt{1+x^2} + C = \sqrt{1+y^2}$
6. $y = \frac{-3x}{1+x} - \frac{3}{1+x} - \frac{x^2}{1+x} + \frac{Ce^x}{1+x}$
7. $\frac{x^4}{4} + xy^3 = C$
8. $\frac{4-y^2}{2(x+1)^2} + \ln|x+1| + \frac{2}{x+1} = \frac{7}{2}$
9. $\frac{(y-x)^2}{y+x} = C$
10. $y = \frac{2}{Ce^{-x} - e^x}$
11. $\tan(x+y) - \sec(x+y) = x + C$
12. Approximately 88.53 milligrams