Laplace Transform Sheet

$L\{f(t)\} = \int_0^\infty f(t)e^{-st}\ dt$

$L\{1\} = \frac{1}{s}$

$L\{t\} = \frac{1}{s^2}$

$L\{t^n\} = \frac{n!}{s^{n+1}}$

$L\{e^{at}\} = \frac{1}{s-a}$

$L\{\sin kt\} = \frac{k}{s^2+k^2}$

$L\{\cos kt\} = \frac{s}{s^2+k^2}$

$L\{\sinh kt\} = \frac{k}{s^2-k^2}$

$L\{\cosh kt\} = \frac{s}{s^2-k^2}$

1st Translation Theorem

$L\{e^{at}f(t)\} = L\{f(t)\}_{s\to s-a} = F(s-a)$

2nd Translation Theorem

$L\{f(t-a)U(t-a)\} = e^{-as}L\{f(t)\} = e^{-as}F(s)$

$L\{g(t)U(t-a)\} = e^{-as}L\{g(t+a)\}$

$L\{U(t-a)\} = \frac{e^{-as}}{s}$

Derivatives of Transforms Theorem

$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} L\{f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$

Transform of a periodic function: Let $f(t)$ be periodic with period $T$. Then

$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\ dt$

Convolution Theorem

$L\{f * g\} = L\{\int_0^t f(\tau)g(t-\tau)\ d\tau\} = L\{f(t)\}L\{g(t)\} = F(s)G(s)$

Transform of a derivative: Let $L\{y(t)\} = Y(s)$. Then

$L\{y^{(n)}(t)\} = s^n Y(s) - s^{(n-1)}y(0) - s^{(n-2)}y'(0) - ... - y^{(n-1)}(0)$

Transform of the Dirac Delta Function

$L\{\delta(t-t_0)\} = e^{-st_0}$