MAP 2302 Additional Problems 2

For 1 – 3, find the general solution of the differential equation:

1. $y''' + 3y'' + 3y' + y = 0$

2. $y^{(4)} + y''' + y'' = 0$

3. $16\frac{d^4y}{dx^4} + 24\frac{d^2y}{dx^2} + 9y = 0$

For 4 and 5, solve the initial value problem:

4. $y'' + 16y = 0, y(0) = 2, y'(0) = -2$

5. $y''' + 12y'' + 36y' = 0, y(0) = 0, y'(0) = 1, y''(0) = -7$

6. Two roots of a cubic auxiliary equation with real coefficients are $m_1 = -\frac{1}{2}$ and $m_2 = 3 + i$. What is the corresponding homogeneous linear differential equation?

For 7 and 8, solve the initial value problem:

7. $5y'' + y' = -6x, y(0) = 0, y'(0) = -10$

8. $\frac{d^2x}{dt^2} + \omega^2 x = F_0 \sin \omega t, x(0) = 0, x'(0) = 0$

9. $xy'' + y' = 0$

10. $x^2y'' + xy' + 4y = 0$

11. $x^2y'' - 3xy' - 2y = 0$

12. $25x^2y'' + 25xy' + y = 0$

13. $x^3y''' - 6y = 0$

14. $xy^{(4)} + 6y''' = 0$

15. $xy'' - 4y' = x^4$

16. $xy'' + y' = x, y(1) = 1, y'(1) = -\frac{1}{2}$

Answers:

1. $y = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}$

2. $y = c_1 + c_2 x + e^{-\frac{x}{2}}\left(c_3 \cos\frac{\sqrt{3}}{2}x + c_4 \sin\frac{\sqrt{3}}{2}x\right)$

3. $y = c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x + c_3 x \cos\frac{\sqrt{3}}{2}x + c_4 x \sin\frac{\sqrt{3}}{2}x$

4. $y = 2\cos 4x - \frac{1}{2}\sin 4x$

5. $y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$

6. $2y''' - 11y'' + 14y' + 10y = 0$

7. $y = -200 + 200e^{-\frac{x}{5}} - 3x^2 + 30x$

8. $x = \frac{F_0}{2\omega^2}\sin\omega t - \frac{F_0}{2\omega}t\cos\omega t$

9. $y = c_1 + c_2 \ln x$

10. $y = c_1 \cos(2\ln x) + c_2 \sin(2\ln x)$

11. $y = c_1 x^{2-\sqrt{6}} + c_2 x^{2+\sqrt{6}}$

12. $y = c_1 \cos\left(\frac{1}{5}\ln x\right) + c_2 \sin\left(\frac{1}{5}\ln x\right)$

13. $y = c_1 x^3 + c_2 \cos\left(\sqrt{2}\ln x\right) + c_3 \sin\left(\sqrt{2}\ln x\right)$

14. $y = c_1 + c_2 x + c_3 x^2 + c_4 x^{-3}$

15. $y = c_1 + c_2 x^5 + \frac{1}{5}x^5 \ln x$

16. $y = \frac{3}{4} - \ln x + \frac{1}{4}x^2$