MAP 2302 Additional Problems 1

Determine a region of the *xy*-plane for which the given differential equation would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

1. $\frac{dy}{dx} = y^{2/3}$

2. $x\frac{dy}{dx} = y$

3. $(4 - y^2)y' = x^2$

4. Determine whether the Existence and Uniqueness Theorem guarantees that the differential equation $y' = \sqrt{y^2 - 9}$ posses a unique solution through the point:

   a. $(1, 4)$ b. $(2, -3)$

5. Solve $\frac{dy}{dx} = e^{3x+2y}$

6. Solve $\frac{dy}{dx} = \frac{xy+3x-y-3}{xy-2x+4y-8}$

7. Solve $x^2\frac{dy}{dx} = y - xy$, $y(-1) = -1$

8. Solve $x^2y' + x(x+2)y = e^x$

9. Solve $\cos x\frac{dy}{dx} + (\sin x)y = 1$

10. Solve $(x+1)\frac{dy}{dx} + y = \ln x$, $y(1) = 10$

11. Find a continuous solution: $\frac{dy}{dx} + 2xy = f(x)$, $y(0) = 2$, where $f(x) = \begin{cases} x, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$

12. Solve $\frac{dy}{dx} = (x+y+1)^2$

13. Solve $\frac{dy}{dx} = \tan^2(x+y)$

Answers:

1. half-planes defined by either $y > 0$ or $y < 0$

2. half-planes defined by either $x > 0$ or $x < 0$

3. the regions defined by $y > 2$, $y < -2$, or $-2 < y < 2$

4. a. yes b. no

5. $-3e^{-2y} = 2e^{3x} + C$

6. $(y+3)^5 e^x = C(x+4)^5 e^y$

7. $y = \frac{e^{-\left(1+\frac{1}{x}\right)}}{x}$

8. $y = \frac{1}{2}x^{-2}e^x + Cx^{-2}e^{-x}$

9. $y = \sin x + C\cos x$

10. $(x+1)y = x\ln x - x + 21$

11. $y = \begin{cases} \frac{1}{2} + \frac{3}{2}e^{-x^2}, & 0 \le x < 1 \\ \left(\frac{1}{2}e + \frac{3}{2}\right)e^{-x^2}, & x \ge 1 \end{cases}$

12. $y = -x - 1 + \tan(x + C)$

13. $2y - 2x + \sin 2(x + y) = C$