The **general (or standard) form of a linear equation** is denoted by **ax + by = c**, where **a**, **b**, and **c** are real numbers, **a** and **b** cannot both be 0.

These are examples of linear equations:

$-4x + 2y = -6.2$

$y = 3.7x - 5$

$y = -3$

These are **not** linear equations:

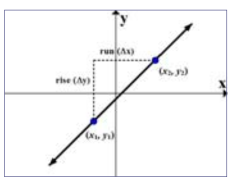
$x^2 + y^2 = 9$

$$\frac{4}{x} + y = 6.5$$

$$y - \sqrt{x} = 7$$

Slope of a Line

Measure of the steepness of a line

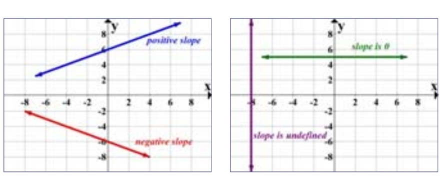


$$m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: *Delta* (uppercase Δ) is the fourth letter of the Greek alphabet, and it is used frequently to represent "change." Δy means "change in y" and Δx means " "change in x."

Slopes of Lines

(Moving from left to right on the graph)



Main Restrictions for Domain

1. Exclude any x-values that result in division by zero.

2. Exclude any x-values that result in even roots of negative numbers. That is, any x-values which make an expression under a square root (or any even root) negative.

Graphs of Functions

The **graph of a function** *f* is the set of all points on the plane of the form (x, *f*(x)).

Vertical Line Test

If a vertical line meets a graph more than once, the graph does *not* represent a function.

For each x-value, there can only be one y-value.

Increasing, Decreasing, and Constant Functions

A function is **increasing** on an interval if it is *rising* as it goes left to right on the interval.
(As x-coordinates increase, y-coordinates increase.)

A function is **decreasing** on an interval if it is *falling* as it goes left to right on the interval.
(As x-coordinates increase, y-coordinates decrease.)

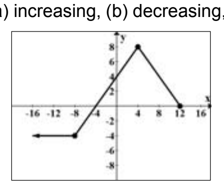
A function is **constant** on an interval if the graph is a *horizontal line* on the interval.
(As x-coordinates increase, y-coordinates remain constant.)

Notes

✓ We read a graph from left to right, and the interval over which a function is increasing, decreasing, or constant is given *only* in terms of the x-values. So, we use interval notation referring to the x-coordinates only.

✓ Use open intervals (parentheses, not brackets) in the interval notation, since the turning/ending points are neither increasing nor decreasing.

Given the graph of y = f(x), determine the intervals for which the function is (a) increasing, (b) decreasing, or (c) constant.



a. The graph rises (increases) from left to right on the interval (–8, 4).

b. The graph falls (decreases) from left to right on the interval (4, 12).

c. The function is constant over the interval (–∞, –8).

Definition of a Linear Function

A **linear function** can be written in the form ***f*(x) = *a*x + *b***, where ***a*** and ***b*** are constants (fixed values).

Examples:

$y = -8x + 2$; $f(x) = -5$; $h(t) = \frac{1}{2} - 9t$ represent linear functions.

$f(x) = x^3 - 2x$; $y = 8^x + 9$; $h(t) = 1/t$ do not represent linear functions.

Linear Functions

Notes:

1. The graph of a linear function is a straight line.
2. The input and output variables can only be raised to the first power.
3. The rate of change (slope) of a linear function *f*(x) = mx + b is given by **m.**
4. A linear function has a constant rate of change.
5. The vertical intercept of a linear function *f*(x) = mx + b is given by **b.**

Slope-Intercept Form

If we know the rate of change (slope), **m**, and the vertical intercept, (0, **b**), of a linear function, we can use the **slope-intercept form, y = mx + b,** to find the equation of the linear function.

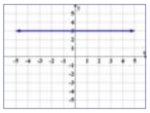
Point-Slope Form

If we know the slope, **m**, and any point (x_1, y_1) on the line, we can use the **point-slope form,** $y - y_1 = m(x - x_1)$ to find the equation of the linear function.

Equations of Horizontal and Vertical Lines

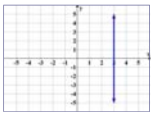
Horizontal Line: y = b, where $\boldsymbol{b}$ is a constant. The line intersects the y-axis at (0, b). Recall that the slope of a horizontal line is 0.

Example: y = 3



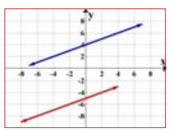
Vertical Line: x = a where $\boldsymbol{a}$ is a constant. The line intersects the x-axis at (a, 0). Recall that the slope of a vertical line is undefined.

Example: x = 3

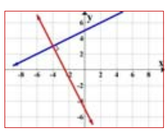


Parallel and Perpendicular Lines

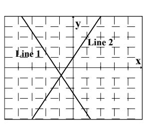
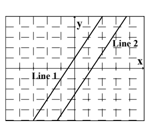
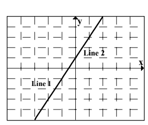
Parallel lines have equal slopes.
Slope of L1 is ½ and slope of L2 is ½.



Perpendicular lines have negative reciprocal slopes.
Slope of L1 is ½ and slope of L2 is –2.



Classification of Linear Systems

One Solution	*No Solution*	*Infinite Number of Solutions*
		
Unique point of intersection	**Parallel lines (no intersection)**	**Lines coincide (infinite points of intersection)**
Consistent (has a solution)	**Inconsistent (has no solution)**	**Consistent (all points are solutions)**
Independent lines	**Independent lines**	**Dependent lines**

Applications involving Break-Even Related Functions

Revenue: Total amount of money received by a business for its products or services.
If a business produces and sells **x** items, R(x) = price • quantity

Cost: Fixed costs (e.g., rent, equipment, etc.) plus Variable costs (e.g., labor, raw materials, utilities, etc.)
C(x) = fixed cost + variable cost

Profit: Amount of money made after all costs and expenses are paid.
P(x) = Revenue – Cost or R(x) – C(x)

Break-even point: Expenses or costs and the revenue are equal; that is, there is neither a profit nor a loss (the profit is 0).
Cost = Revenue or C(x) = R(x)

Solving a Linear System with Two Variables by Substitution

(1) Choose one of the equations and solve it for one of the variables in terms of the other.

(2) Substitute the expression from step (1) into the other equation. This will result in an equation with just one variable. Solve for the variable.

(3) Substitute the value found in step (2) into either of the original equations to find the remaining variable.

(4) Write the answer as an ordered pair and check the solution to the system in both original equations.

Note: When solving an application problem, state the answer in the context of the problem.

Solving a Linear System with Two Variables by Elimination

(1) Eliminate one of the variables by using the concept of "opposites." If necessary, multiply one or both equations by a nonzero constant that will produce coefficients of either **x** or **y** that are opposites of each other.

(2) Add the two equations, making sure that one of the variables drops out, leaving one equation and only one unknown.

(3) Solve the resulting equation for the variable.

(4) Back-substitute the value found in (3) into one of the original equations to find the value of the remaining variable.

(5) Write the answer as an ordered pair and check the solution to the system in both original equations.

Note: When solving an application problem, state the answer in the context of the problem.

Linear Inequalities

A linear inequality in two variables can be written in one of the following forms:

$$ax + by > c$$
$$ax + by < c$$
$$ax + by \geq c$$
$$ax + by \leq c$$

where a, b, and c are real numbers, and a and b are not both zero.

An ordered pair (x, y) will be a **solution of a linear inequality in two variables** if it satisfies the inequality.

Graphical Solution

(1) Replace the inequality symbol with an equal sign and graph the line, which we call the "boundary line."

⇨If the inequality symbol is ≥ or ≤, the line will be part of the solution set. Draw a *solid line* to include the line in the solution.

⇨ If the inequality symbol is > or <, the line will **not** be part of the solution set. Draw a *dashed line* to exclude the line.

(2) Select any point ("test point") that does not lie on the line to determine the **region** whose points will satisfy the inequality.

⇨ If the point (0, 0) is not on the line, using it as a test point is convenient for calculations.

(3) If the test point satisfies the inequality, shade the region that contains the point. If it *does not* satisfy the inequality, then shade the opposite region.

⇨ The shaded region will contain all the points that satisfy the inequality.

System of Linear Inequalities

A **system of linear inequalities in two variables** is a set of two or more linear inequalities to be solved together.

The solution set will consist of all ordered pairs (x, y) that satisfy the inequalities simultaneously.

We find the solution of the system by graphing each linear inequality and identifying where the shaded regions overlap.

Solving a System of Linear Inequalities in Two Variables

(1) Solve each of the inequalities for **y**.

(2) For each inequality, replace the inequality symbol with an equal sign and graph the line (boundary line).
⇨ For ≥ or ≤, draw a *solid line* to include the line in the solution.
⇨ For > or <, draw a *dashed line* to exclude the line.

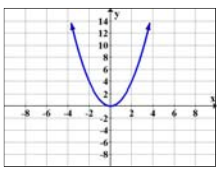
(3) Shade the appropriate regions on all inequalities.

(4) The solution of the system will be the shaded region where all the graphs overlap. If the shaded regions do not overlap, then the system has no solutions.

(5) Use test points from all regions to verify the result.

(6) State any vertices.

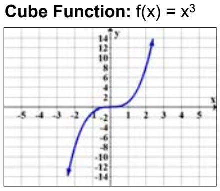
Square Function: $f(x) = x^2$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

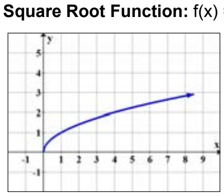
Cube Function: $f(x) = x^3$



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

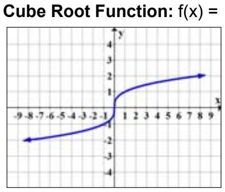
Square Root Function: $f(x) = \sqrt{x}$



Domain: $[0, \infty)$

Range: $[0, \infty)$

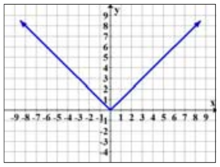
Cube Root Function: $f(x) = \sqrt[3]{x}$



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

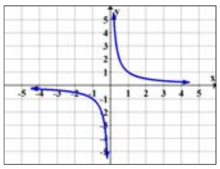
Absolute Value Function: f(x) = |x|



Domain: (–∞, ∞)

Range: [0, ∞)

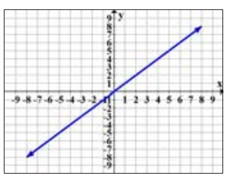
Reciprocal Function: f(x) = $\frac{1}{x}$



Domain: (–∞, 0) U (0, ∞)

Range: (–∞, 0) U (0, ∞)

Identity Function: f(x) = x

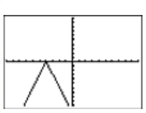


Domain: (–∞, ∞)

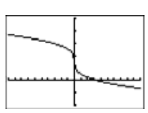
Range: (–∞, ∞)

Each of the following graphs is a family member of one of the basic functions. Determine the basic function in each case.

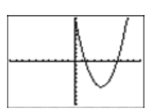
a.



b.



c.



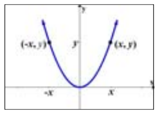
a. Absolute function

b. Cube root function

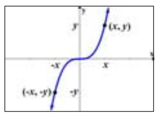
c. Square function (also known as "Parabola")

Even and Odd Functions

Even Function: Graph is symmetric about the **y-axis**.
If (x, y) is a point on the graph, then (–x, y) is also a point on the graph.
Example:



Odd Function: Graph is symmetric about the **origin**.
If (x, y) is a point on the graph, then (–x, –y) is also a point on the graph.
Example:



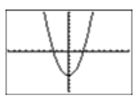
Algebraic Test for Even Function

$f(-x) = f(x)$
(Replacing x with –x results in the same function.)

a. Determine if $f(x) = x^2 - 6$ is an even function.

$f(-x) = (-x)^2 - 6$
$= x^2 - 6$

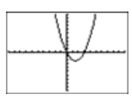
Since $f(-x) = f(x)$, this is an even function.
Observe graph is symmetric about the y-axis:



b. Determine if $f(x) = x^2 - 3x$ is an even function.

$f(-x) = (-x)^2 - 3(-x)$
$= x^2 + 3x$

Since $f(-x) \neq f(x)$, this is not an even function.
Graph is *not* symmetric about the y-axis:



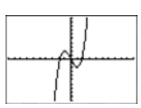
Algebraic Test for Odd Function

$f(-x) = -f(x)$
(Replacing x with –x gives the negative of the original function.)

a. Determine if $f(x) = x^3 - 3x$ is an odd function.

$f(-x) = (-x)^3 - 3(-x)$
$= -x^3 + 3x$

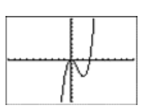
Since $f(-x) = -f(x)$, this is an odd function.
Observe graph is symmetric about the origin:



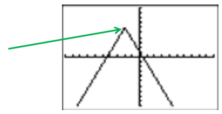
b. Determine if $f(x) = x^3 - 3x^2$ is an odd function.

$f(-x) = (-x)^3 - 3(-x)^2$
$= -x^3 - 3x^2$

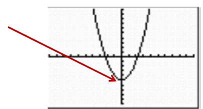
Since $f(-x) \neq -f(x)$, this is not an odd function.
Graph is *not* symmetric about the origin:



Global or Absolute Maximum of a Function
Highest point over the entire domain of a function.



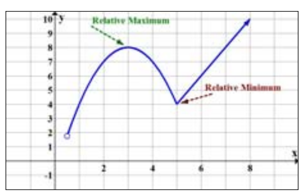
Global or Absolute Minimum of a Function
Lowest point over the entire domain of a function.



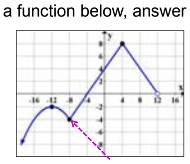
Relative Maximum and Relative Minimum

Turning Points: Where a graph of a function changes behavior from increasing to decreasing or vice versa; highest or lowest points at specific intervals of the graph.

Each turning point is called a Relative (or local) Maximum or a Relative (or local) Minimum.



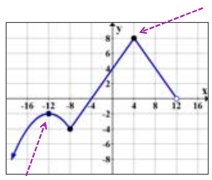
Given the graph of a function below, answer the questions.



a. At what value of **x** is the relative minimum occurring?
The graph has a turning point where it changes its behavior from a decreasing interval to an increasing interval, when x = –8. Therefore, this function has a relative minimum at x = –8.

b. What is the relative minimum?
The relative minimum is the output of the function at x = –8, or f(–8) = –4.
The relative minimum in ordered pair form would be (–8, –4).

(Contd.)



c. For what values of **x** does the function have relative maxima?

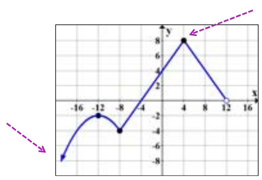
The graph has two turning points where it changes its behavior from an increasing interval to a decreasing interval at x-values of –12 and 4.

This function has relative maxima at x = –12 and at x = 4.

d. What are the relative maxima?

The relative maxima are f(–12) = –2 and f(4) = 8.

(Contd.)



e. State any absolute maximum or minimum on this graph.

The highest point over the entire domain of the function is the point (4, 8).

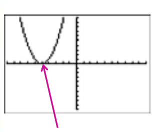
The function will have an absolute maximum of 8 at x = 4.

There is no absolute minimum.

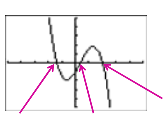
Zeros of a Function

The x-intercept of the graph of a function is also called the **zero** of the function.

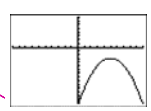
A function can have no zero, one zero, or multiple zeros.



One zero



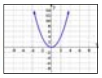
Three zeros



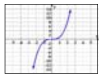
No zero

Review of 7 Basic Functions

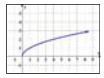
Square
$f(x) = x^2$



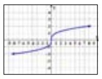
Cube
$f(x) = x^3$



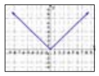
Square Root
$f(x) = \sqrt{x}$



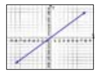
Cube Root
$f(x) = \sqrt[3]{x}$



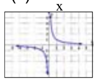
Absolute Value
$f(x) = |x|$



Identity
$f(x) = x$



Reciprocal
$f(x) = \frac{1}{x}$



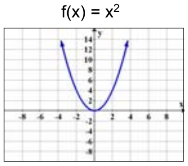
Vertical Shifts (Up)

For a function $y = f(x)$ and $c > 0$:

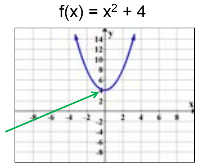
The graph of $y = f(x) + c$ is equivalent to the graph of $f(x)$ shifted upward c units.

Example:

$f(x) = x^2$



$f(x) = x^2 + 4$



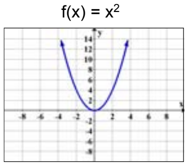
Vertical Shifts (Down)

For a function $y = f(x)$ and $c > 0$:

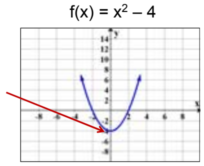
The graph of $y = f(x) - c$ is equivalent to the graph of $f(x)$ shifted downward c units.

Example:

$f(x) = x^2$



$f(x) = x^2 - 4$



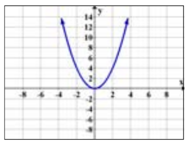
Horizontal Shifts (left)

For a function y = f(x) and c > 0:

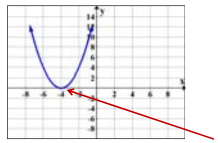
The graph of y = f(x + c) is equivalent to the graph of f(x) shifted left c units.

Example:

$f(x) = x^2$



$f(x) = (x + 4)^2$



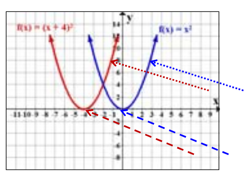
Horizontal Shifts: If y = f(x + c), why shift left?

Examine a table of values for each function:

x	–4	–3	–2	–1	0	1	2
$f(x) = x^2$	16	9	4	1	0	1	4

x	–4	–3	–2	–1	0	1	2
$f(x) = (x + 4)^2$	0	1	4	9	16	25	36

Notice that every point on the graph of $f(x) = (x + 4)^2$ is 4 units to the left of a corresponding point on the graph of $f(x) = x^2$.



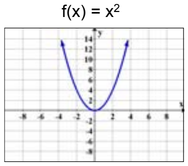
Horizontal Shifts (right)

For a function y = f(x) and c > 0:

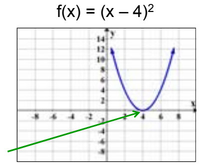
The graph of y = f(x – c) is equivalent to the graph of f(x) shifted right c units.

Example:

$f(x) = x^2$



$f(x) = (x – 4)^2$



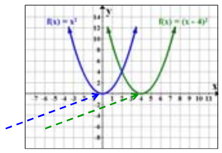
Horizontal Shifts: If y = f(x – c), why shift right?

Examine a table of values for each function:

x	–4	–3	–2	–1	0	1	2
$f(x) = x^2$	16	9	4	1	0	1	4

x	–4	–3	–2	–1	0	1	2
$f(x) = (x – 4)^2$	64	49	36	25	16	9	4

Observe that every point on the graph of $f(x) = (x – 4)^2$ is 4 units to the right of a corresponding point on the graph of $f(x) = x^2$.



Summary of Vertical and Horizontal Translations

The location of the graph of the function is changed, but the size and shape of the graph is not changed.

Vertical shifts: Adding or subtracting "*outside*" the function
Horizontal shifts: Adding or subtracting "*inside*" the function

f(x) + c	**Shift graph of f(x) upward c units**
f(x) − c	**Shift graph of f(x) downward c units**
f(x + c)	**Shift graph of f(x) left c units**
f(x − c)	**Shift graph of f(x) right c units**

Vertical and Horizontal Translations

Example:

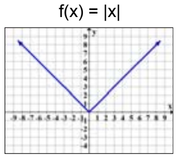
f(x) = |x – 2| + 5

inside *outside*

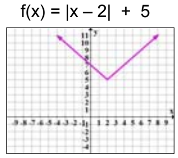
move opposite direction *move same direction*

"2 units right, 5 units up"

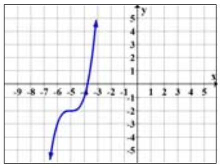
f(x) = |x|



f(x) = |x – 2| + 5



Identify the following graph as a translation of one of the basic functions, and write the equation for the graph.



This is the graph of $f(x) = x^3$ shifted left 5 units and down 2 units.

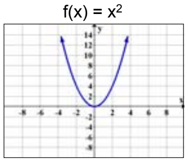
$f(x) = (x + 5)^3 – 2$

Reflections about the x-axis

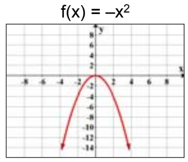
The graph of $y = –f(x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the **x**-axis.

Example:

$f(x) = x^2$



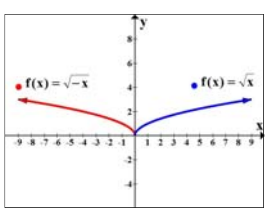
$f(x) = –x^2$



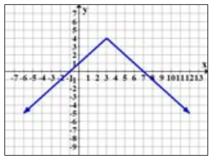
Reflections about the y-axis

The graph of $y = f(–x)$ is obtained by reflecting the graph of $y = f(x)$ with respect to the **y**-axis.

Example:



Given the graph below, (a) state the basic function, (b) describe the transformations applied, and (c) write the equation for the given graph.



a. $f(x) = |x|$

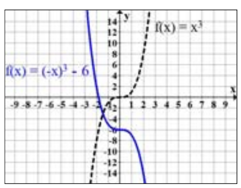
b. Reflection about the x-axis
Horizontal shift: 3 units right
Vertical shift: 4 units upward

c. $f(x) = -|x-3| + 4$

Use the corresponding basic function to construct the graph of the function f(x) = $(-x)^3 - 6$.

The basic function is f(x) = x^3.

Transformations in f(x) = $(-x)^3 - 6$:
Reflection about the y-axis
Vertical shift of 6 units downward



Stretching and Compressing

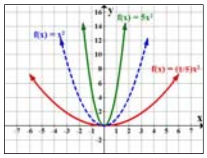
Let **c** be a constant. The graph of y = **c**f(x) is obtained by vertically stretching or compressing the graph of y = f(x).

✓ If |**c**| > 1, the graph will be vertically stretched by a factor of **c**.

✓ If 0 < |**c**| < 1, the graph will be vertically compressed by a factor of **c**.

Stretching and Compressing

Example:



In $f(x) = 5x^2$, stretching "elongates" the graph; the transformed graph appears "narrower" than the graph of $f(x) = x^2$.

In $f(x) = (1/5)x^2$, compressing "flattens" the graph; the transformed graph appears "wider" than the graph of $f(x) = x^2$.

Combining Multiple Transformations to Sketch a Graph

In general, it may be useful to use the following order:

1. Reflection
2. Vertical stretch or compression
3. Horizontal shift
4. Vertical shift

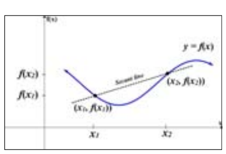
Other possible ordering will produce the same result, but do the vertical shift last.

Average Rate of Change

Let **f** be a function. Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ represent two distinct points on the graph of **f**, where $x_1 \neq x_2$.

The **average rate of change** of **f** between x_1 and x_2 is given by

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

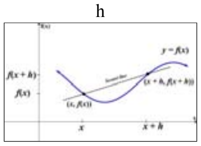


The Difference Quotient

Let **f** be a function and **h** a nonzero constant. Let (x, f(x)) and (x + h, f(x + h)) represent two distinct points on the graph of **f**.

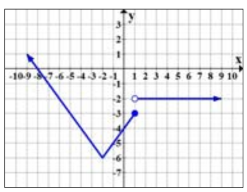
The average rate of change of **f** between **x** and **x + h** is the **difference quotient**, given by

$$\frac{f(x+h)-f(x)}{h}$$



Notice that the **main** operations involved in the "difference quotient" are subtraction (*difference*) and division (*quotient*).

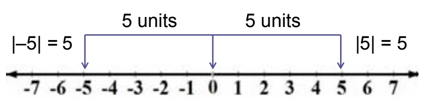
Find the piecewise-defined function whose graph is shown.



$$f(x)=\begin{cases}|x+2|-6 & \text{if} \quad x\le 1\\ -2 & \text{if} \quad x>1\end{cases}$$

Review of Absolute Value

The absolute value of a real number **x**, denoted by **|x|**, describes the distance of x from 0 on the number line, regardless of direction. For example, |x| = 5 has two solutions: x = –5 or x = 5.



Solving Absolute Value Equations

✓If **x** represents an algebraic expression and $a > 0$, then $|x| = a$ will have two solutions:

$$x = a \quad \text{or} \quad x = -a$$

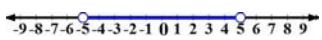
✓If $a = 0$, then $|x| = a$ has only the solution $x = 0$.

✓If $a < 0$, then $|x| = a$ will have no real solution, since the absolute value of any quantity is never negative.

Note:
Make sure that the absolute value expression has been isolated on one side of the equation before solving.

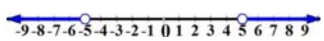
Inequalities: Review of Absolute Value

$|x| < 5$ means that the distance of **x** from 0 is *less than* 5 units.



The solution for $|x| < 5$ can be expressed as $-5 < x < 5$.
Using interval notation: $(-5, 5)$.

$|x| > 5$ means that the distance of **x** from 0 is *more than* 5 units.



The solution for $|x| > 5$ can be expressed as $x < -5$ or $x > 5$.
Using interval notation $(-\infty, -5) \cup (5, \infty)$.

Solving Absolute Value Inequalities

If **x** represents an algebraic expression and $a > 0$,

✓ $|x| < a$ if and only if $-a < x < a$.

✓ $|x| > a$ if and only if $x < -a$ or $x > a$.

✓ $|x| \le a$ if and only if $-a \le x \le a$.

✓ $|x| \ge a$ if and only if $x \le -a$ or $x \ge a$.

Definition of Quadratic Equation

A **quadratic equation** in **x** is a second-degree equation that has the standard form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, $a \neq 0$.

a: leading coefficient

b: coefficient of the first-degree term

c: constant term

Solving Quadratic Equations by Factoring

Zero Product Property:
If a and b are real numbers, and $ab = 0$, then either $a = 0$, or $b = 0$, or both a and b are zero.

Factoring Method:
1. Write the equation in standard form. (Make sure one side of the equation is 0.)
2. Factor the nonzero side of the equation.
3. Apply the zero-product property. (Set each factor = 0.)
4. Solve for the variable.
5. Verify your answers in the original equation.

Solving by the Square Root Method

Square Root Method: Useful method when $b = 0$ in $ax^2 + bx + c = 0$ (that is, no presence of the first-degree term, bx, in the standard form).

Square Root Property:
If $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$ for k constant.
Solutions of quadratic equations of the form $x^2 = k$ are given by: $x = \pm\sqrt{k}$.

(continued on the next slide)

Solving by the Square Root Method

1. Isolate the squared variable term.

2. Apply the square root property to undo the square. Remember to insert "±" on the numeric side of the equation, since we want both the positive and negative square roots.

3. Simplify the radical expression.

4. Verify your answers in the original equation.

Quadratic Formula

The solutions of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $\boldsymbol{a}$ is the coefficient of x^2, $\boldsymbol{b}$ is the coefficient of x, and $\boldsymbol{c}$ is the constant term of the quadratic equation.

The Discriminant

How can we determine the nature of the solutions of a quadratic equation?

The **discriminant** of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, is given by $\boldsymbol{b^2 - 4ac}$.
(Note: This expression is equivalent to the radicand in the quadratic formula.)

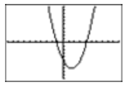
- ✓ If $b^2 - 4ac > 0$, the equation has two distinct real solutions.
- ✓ If $b^2 - 4ac = 0$, the equation has one real solution.
- ✓ If $b^2 - 4ac < 0$, the equation has no real solutions.

Graphical Methods

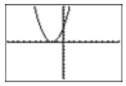
The solutions of a quadratic equation $ax^2 + bx + c = 0$, are the **x-intercepts** of the graph of the equation. If the graph has no x-intercepts, then the equation has no real solutions.

Examples:

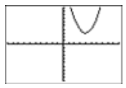
Two distinct real solutions:



One real solution:



No real solution:



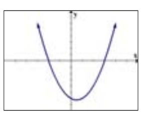
Definition of Quadratic Function

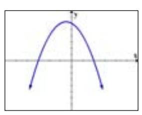
A **quadratic function** is a function of the form

$$f(x) = ax^2 + bx + c$$

where a, b, and c are real numbers, $a \neq 0$.

The graph of a quadratic function resembles a U-shaped curve and it is called a ***parabola***. It will have either an upward or a downward concavity.





$y = ax^2 + bx + c$ is frequently called the *general form* of a parabola.

Features of a Parabola

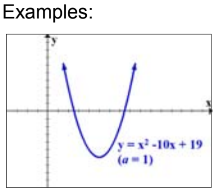
Concavity:

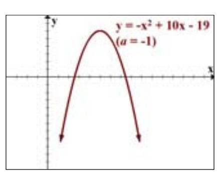
In a parabola of the form $y = ax^2 + bx + c$, $a \neq 0$:

✓ If $a > 0$, the parabola is concave up.

✓ if $a < 0$, it is concave down.

Examples:



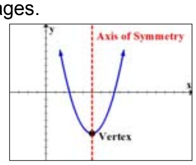


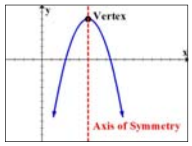
Features of a Parabola (contd.)

Vertex: “Turning point" of the graph of the quadratic function.

- ✓ If concave up, vertex is the lowest or *minimum* point.
- ✓ If concave down, vertex is the highest or *maximum* point.

Axis of symmetry: Vertical line through the vertex; divides the parabola into two parts which are mirror images.





Vertex of a Parabola of the Form f(x) = $ax^2 + bx + c$, $a \neq 0$.

✓x-coordinate:
The x-coordinate of the vertex is given by $x = -\frac{b}{2a}$.

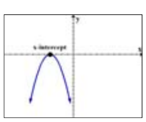
✓y-coordinate:
The y-coordinate of the vertex is given by $f\left(-\frac{b}{2a}\right)$.

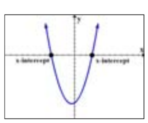
Summary:
First, find the x-coordinate of the vertex by using $x = -\frac{b}{2a}$.

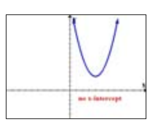
Then, evaluate the quadratic function at this x-value to find the y-coordinate of the vertex.

Features of a Parabola (contd.)

X-intercept: The graph of a parabola may cross the x-axis once, or at two different points, or not at all.







If they exist, the x-intercepts of the parabola will occur when y = 0.

So, they are found by setting $ax^2 + bx + c = 0$ and solving for **x.**

(Solve by factoring, if possible, or applying one of the various solving methods learned in Section 4.1.)

Features of a Parabola (contd.)

Y-intercept: The graph of a parabola crosses the y-axis exactly once.

The y-intercept is found by letting x = 0 in $y = ax^2 + bx + c$, and solving for **y**.

Observe when x = 0, we have

$$y = a(0)^2 + b(0) + c = 0$$

So, in a parabola $y = ax^2 + bx + c$, the y-intercept is given by $y = c$; in ordered pair form $(0, c)$.

Graphing a Parabola of the Form $y = ax^2 + bx + c$, $a \neq 0$.

1. Determine the concavity:
If $a > 0$, the parabola is concave up; if $a < 0$, concave down

2. Find the vertex: $(-b/2a, f(-b/2a))$
✓If concave up, the vertex is a minimum point
✓If concave down, the vertex is a maximum point

3. Identify the y-intercept: $f(0)$ is given by the value of c.

4. Find any x-intercepts: Let y = 0 and solve for x.

5. Plot the vertex, the y-intercept, and x-intercepts (if any), and connect them with a smooth U-shaped curve. Show the axis of symmetry. (If needed, use the axis of symmetry to plot a symmetric point to the y-intercept to complete graph.)

Vertex Form of a Quadratic Function

$f(x) = a(x - h)^2 + k$, $a \neq 0$, where (h, k) are the coordinates of the vertex.

$a > 0$, concave up; $a < 0$, concave down
If $|a| < 1$, graph will be *wider* than the graph of $y = x^2$.
If $|a| > 1$, graph will be *narrower* than the graph of $y = x^2$.

The axis of symmetry has the equation $x = h$.

The h-coordinate of the vertex represents a horizontal shift and it is the "opposite" of the value stated in $f(x) = a(x - h)^2 + k$.

The value of the k-coordinate of the vertex represents a vertical shift, and it will have the same sign as the one in the given function.