

**Geometric Series:**  $\sum_{n=1}^{\infty} ar^{n-1}$  converges to  $\frac{a}{1-r}$  if  $|r| < 1$ .

**Divergence Test:** If  $\lim_{n \rightarrow \infty} a_n$  does not exist or if  $\lim_{n \rightarrow \infty} a_n \neq 0$ , then the series  $\sum_{n=1}^{\infty} a_n$  is divergent.

**Integral Test:** Suppose  $f$  is a continuous, positive, decreasing function on  $[1, \infty)$  and let  $a_n = f(n)$ . Then the series  $\sum_{n=1}^{\infty} a_n$  is convergent if and only if the improper integral  $\int_1^{\infty} f(x) dx$  is convergent.

**P series Test:** The  $p$ -series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is convergent if  $p > 1$  and divergent if  $p \leq 1$ .

**Comparison Test:** Suppose that  $\sum a_n$  and  $\sum b_n$  are series with positive terms.  
If  $\sum b_n$  is convergent and  $a_n \leq b_n$  for all  $n$ , then  $\sum a_n$  also converges.  
If  $\sum a_n$  is divergent and  $a_n \leq b_n$  for all  $n$ , then  $\sum b_n$  also diverges.

**Limit Comparison** Suppose that  $\sum a_n$  and  $\sum b_n$  are series with positive terms. If

**Test:**  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$  where  $c$  is a finite number and  $c > 0$ , then either both series converge or both series diverge.

**Alternating Series** If the alternating series  $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$  satisfies

- Test:**
- 1)  $b_{n+1} \leq b_n$  for all  $n$  and
  - 2)  $\lim_{n \rightarrow \infty} b_n = 0$  then the series converges.

**Ratio Test:** Suppose that  $\sum a_n$  is a series with positive terms. If  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L < 1$

then the series converges. If  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = L > 1$  then the series diverges.

**Root Test:** Suppose that  $\sum a_n$  is a series with positive terms. If  $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = L < 1$   
then the series converges. If  $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = L > 1$  then the series diverges.

If a series  $\sum a_n$  converges and  $\sum |a_n|$  converges, then  $\sum a_n$  is **absolutely convergent**.

If a series  $\sum a_n$  converges and  $\sum |a_n|$  diverges, then  $\sum a_n$  is **conditionally convergent**.

**MacLaurin Series:**  $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

**Taylor Series:**  $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$