MAC 2233 Marginal Analysis Worksheet

1. (Armstrong & Davis, section 3.2 problem 13) The Country Day Company determines that the daily cost of producing lawn tractor tires can be modeled by $C(x) = 100 + 40x - 0.001x^2, 0 \leq x \leq 300$ where x represents the number of tires produced each day and $C(x)$ is the total cost, in dollars, of producing the tires.
   a. Determine $C'(x)$, the marginal cost function.
   b. Evaluate and interpret $C'(200)$.
2. (Armstrong & Davis, section 3.2 problem 17) For the cost function in the preceding problem,
   a. Determine $\bar{C}(x)$, the average cost function. Evaluate and interpret $\bar{C}(200)$.
   b. Determine $\bar{C}'(x)$, the marginal average cost function. Evaluate and interpret $\bar{C}'(200)$.
3. (Tan, section 3.4 problem 5) Custom Office makes a line of executive desks. It is estimated that the total cost for making x units of their Senior Executive model is $C(x) = 100x + 200,000$ dollars per year.
   a. Find the average cost function, $\bar{C}(x)$.
   b. Find the marginal average cost function, $\bar{C}'(x)$.
   c. What happens to $\bar{C}(x)$ when x is very large? Interpret your results.
4. (Armstrong & Davis, section 3.2 problem 21) A telemarketer determines that the monthly profit from selling magazine subscriptions can be modeled by $P(x) = 5x + x^{1/2}, 0 \leq x \leq 100$ where x is the number of magazine subscriptions sold per month and $P(x)$ is the profit in dollars.
   a. Determine $P'(x)$, the marginal profit function.
   b. Evaluate and interpret $P'(55)$.
5. (Armstrong & Davis, section 3.2 problem 23) For the profit function in the preceding problem,
   a. Determine $\bar{P}(x)$, the average profit function. Evaluate and interpret $\bar{P}(55)$.
   b. Determine $\bar{P}'(x)$, the marginal average profit function. Evaluate and interpret $\bar{P}'(55)$.
6. (Tan, section 3.4 problem 13) The weekly demand for the Pulsar 25 color console television is $p = 600 - 0.05x$ $(0 \leq x \leq 12,000)$ where p denotes the wholesale unit price in dollars and x denotes the quantity demanded. The weekly total cost function associated with manufacturing the Pulsar 25 is given by $C(x) = 0.000002x^3 - 0.03x^2 + 400x + 80,000$ where $C(x)$ denotes the total cost incurred in producing x sets.
   a. Find the revenue function R and the profit function P.
   b. Find the marginal cost function C', the marginal revenue function R', and the marginal profit function P'.
   c. Compute $C'(2000)$, $R'(2000)$, and $P'(2000)$ and interpret your results.
7. (Tan, section 3.4 problem 15) Find the average cost function associated with the cost function of the preceding exercise.
   a. What is the marginal average cost function, $\bar{C}'$?
   b. Compute $\bar{C}'(5000)$ and $\bar{C}'(10,000)$ and interpret your results.

8. (Armstrong & Davis, section 3.2 problem 27) The NewJoy Company hires a consulting firm to audit their books and consequently revise their price and cost functions to $p(x) = 23$ and $C(x) = \frac{x^2}{95} + \frac{7}{2}x + 5500$.
   a. Algebraically derive the profit function P and simplify it.
   b. Evaluate $P(500)$ and interpret.
   c. Evaluate $P'(500)$ and interpret.

9. (Armstrong & Davis, section 3.2 problem 29) The Vroncom Company determines that the price-demand function for their handheld computer device is $p(x) = -\frac{x}{30} + 300$. They know that their fixed costs are \$150,000 and variable cost is 30 dollars per device.
   a. Determine the revenue function and the cost function.
   b. Determine the profit function. Find the smallest and largest production levels x so that the company realizes a profit.
   c. Evaluate $P'(1000)$ and interpret.

10. (Tan, section 3.4 problem 17) The quantity of Sicard wristwatches demanded each month is related to the unit price by the equation $p = \frac{50}{0.01x^2+1}$ $(0 \le x \le 20)$ where p is measured in dollars and x in units of a thousand.
   a. Find the revenue function R.
   b. Find the marginal revenue function R'.
   c. Compute $R'(2)$ and interpret your results.

11. (Armstrong & Davis, section 4.1 problem 73) The StopCop Company determines that the cost to produce auto antitheft devices is modeled by $C(x) = (3x + 6)^{1.5} + 30, 0 \le x \le 50$ where x represents the number of auto antitheft devices produced in hundreds and $C(x)$ represents the production costs in thousands of dollars.
   a. Determine the marginal cost function.
   b. Evaluate and interpret $C'(5)$.

12. (Armstrong & Davis, section 4.1 problem 79) The HotSpark Company has assumed that the price-demand function for their spark plug is $p(x) = \frac{125}{\sqrt{2x+5}}, 0 \le x \le 20$ where x represents the number of spark plugs manufactured in hundreds and $p(x)$ is the price of the spark plug.
   a. Determine $p'(x)$.
   b. Evaluate $p'(20)$ and interpret.
   c. Determine the revenue function.
   d. Determine the marginal revenue function.
   e. Evaluate and interpret $R'(20)$.