MAC 2233 Definite Integral Worksheet

1. (Armstrong & Davis, section 6.4 problem 13) The marginal revenue function for the FrontRide Bus Company is given by \( R'(x) = 0.000045x^2 - 0.03x + 3.75, 0 \leq x \leq 500 \) where \( x \) represents the number of bus tickets sold and \( R'(x) \) is the marginal revenue measured in dollars per ticket. Evaluate \( \int_0^{200} R'(x) \, dx \) and interpret.

   The revenue generated from selling the first 200 tickets is $270.

2. (Armstrong & Davis, section 6.4 problem 15) The ScandiTrac Company determines that their marginal profit function for producing and selling a new economy model of cross-country ski machine at a mall is given by \( P'(x) = 0.3x^2 + 0.2x, 0 \leq x \leq 30 \) where \( x \) is the number of machines produced and sold and \( P'(x) \) is the marginal profit measured in dollars per ski machine. Evaluate \( \int_0^{20} P'(x) \, dx \) and interpret.

   As production and sales increase from 10 to 20 ski machines, the total increase in profit is $730.

3. (Armstrong & Davis, section 6.5 problem 63) The CustomKey Company determines that the marginal cost of producing sterling silver key fobs is given by \( C'(x) = \frac{10x}{\sqrt{x^2+10000}}, 0 \leq x \leq 100 \) where \( x \) represents the number of fobs produced daily and \( C'(x) \) is the marginal cost in dollars.
   
   a. Evaluate \( C'(75) \) and interpret.

      When daily production is 75 fobs, it costs about $6 to produce the 76th fob.

   b. Evaluate \( \int_0^{75} C'(x) \, dx \) and interpret.

      The total increase in daily cost of producing from 0 to 75 fobs is $250.

   c. Knowing that the cost to produce 75 fobs is $3250, recover the cost function \( C \).

      \[ C(x) = 10\sqrt{x^2 + 10,000} + 2000 \]

   d. Determine the fixed costs.

      $3000

4. (Armstrong & Davis, section 6.6 problem 25) The Top-2-Bottom Dress Store has a clearance sale to remove old inventory. Past records show that the rate of sales is given by \( S(t) = \frac{40}{t}, 1 \leq t \leq 8 \) where \( t \) represents the number of days that the sale has been running and \( S(t) \) represents the sales rate, measured in dresses per day.

   a. Evaluate \( S(2) \) and interpret.

      The sales rate after 2 days is 20 dresses per day.

   b. Determine the total increase in the number of dresses sold between days 2 and 4.

      \[ 40 \ln 2 \approx 28 \]

5. (Armstrong & Davis, section 6.6 problem 59) The FlowStop Company determines that the marginal cost for their new line of faucet parts is given by \( C'(x) = 1.50 + 0.04e^{0.02x} \).

   a. Evaluate \( C'(20) \) and interpret.

      At a production level of 20 faucets, the cost to produce the 21st faucet is about $1.56.

   b. If the number of units of the part produced increases from 100 to 150, what is the total increase in cost?

      $100.39
6. (Waner & Costenoble, section 6.4 problem 55) The rate of U.S. sales of bottled water for the period 1993-2003 could be approximated by \( R(t) = 17t^2 + 100t + 2300 \) million gallons per year \( (3 \leq t \leq 13) \) where \( t \) is the time in years since 1990. Estimate the total U.S. sales of bottled water from 1995 to 2000. (Round your answer to the nearest billion gallons.)

20 billion gallons