Chapter 5

THE STANDARD DEVIATION AS A RULER AND THE NORMAL MODEL

1

STANDARDIZING WITH Z-SCORES

- We compare individual data values to their mean, relative to their standard deviation using the following formula:

$$z = \frac{(y - \bar{y})}{s}$$

- We call the resulting values standardized values, denoted as *z*. They can also be called *z*-scores.

2

Z-SCORE

- Written out, that is

$$z = \frac{\text{observed value} - \text{mean}}{\text{standard deviation}}$$

3

STANDARDIZING WITH Z-SCORES

- Standardized values have no units.
- *z*-scores measure the distance of each data value from the mean in standard deviations.
- A negative *z*-score tells us that the data value is *below* the mean, while a positive *z*-score tells us that the data value is *above* the mean.

4

BENEFITS OF STANDARDIZING

- Standardized values have been converted from their original units to the standard statistical unit of *standard deviations from the mean*.
- Thus, we can compare values that are measured on different scales, with different units, or from different populations.

5

WHEN IS A Z-SCORE BIG?

- A *z*-score gives us an indication of how unusual a value is because it tells us how far it is from the mean.
- Remember that a negative *z*-score tells us that the data value is *below* the mean, while a positive *z*-score tells us that the data value is *above* the mean.
- The larger a *z*-score is (negative or positive), the more unusual it is.

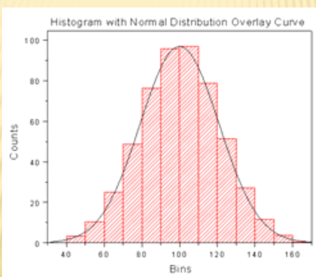
6

WHEN IS A Z-SCORE BIG? (CONT.)

- There is no universal standard for *z*-scores, but there is a model that shows up over and over in Statistics.
- This model is called the Normal model (You may have heard of "bell-shaped curves.").
- Normal models are appropriate for distributions whose shapes are unimodal and roughly symmetric.
- These distributions provide a measure of how extreme a *z*-score is.

7

NORMAL DISTRIBUTION



Retrieved from http://www.originlab.com/www/resources/graph_gallery/images_galleries/Histo.gif, January 27, 2010.

8

WHEN IS A Z-SCORE BIG? (CONT.)

- There is a Normal model for every possible combination of mean and standard deviation.
  - We write $N(\mu,\sigma)$ to represent a Normal model with a mean of μ and a standard deviation of σ.
- We use Greek letters because *this* mean and standard deviation do not come from data—they are numbers (called parameters) that specify the model.

9

WHEN IS A Z-SCORE BIG? (CONT.)

- Summaries of data, like the sample mean and standard deviation, are written with Latin letters. Such summaries of data are called statistics.
- When we standardize Normal data, we still call the standardized value a *z*-score, and we write

$$z = \frac{y - \mu}{\sigma}$$

10

WHEN IS A Z-SCORE BIG? (CONT.)

- Once we have standardized, we need only one model:
  - The *N*(0,1) model is called the standard Normal model (or the standard Normal distribution).
- Be careful—don't use a Normal model for just any data set, since standardizing does not change the shape of the distribution.

11

WHEN IS A Z-SCORE BIG? (CONT.)

- When we use the Normal model, we are assuming the distribution is Normal.
- We cannot check this assumption in practice, so we check the following condition:
  - Nearly Normal Condition: The shape of the data's distribution is unimodal and symmetric.
  - This condition can be checked with a histogram or a Normal probability plot (explained in text).

12

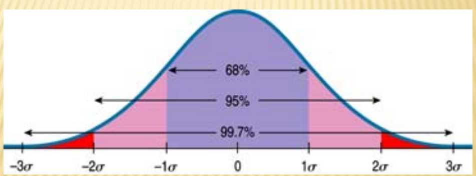
THE 68-95-99.7 RULE

- It turns out that in a Normal model:
  - about 68% of the values fall within one standard deviation of the mean;
  - about 95% of the values fall within two standard deviations of the mean; and,
  - about 99.7% (almost all!) of the values fall within three standard deviations of the mean.

13

THE 68-95-99.7 RULE (CONT.)

- The following shows what the 68-95-99.7 Rule tells us:



From *Stats Modeling the World* by Bock, Velleman, & De Veaux, 2010, p. 113.

14

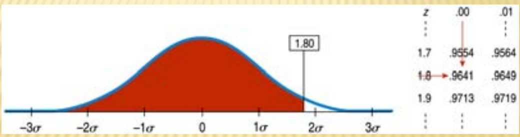
FINDING NORMAL PERCENTILES BY HAND

- When a data value doesn't fall exactly 1, 2, or 3 standard deviations from the mean, we can look it up in a table of Normal percentiles.
- Table Z in Appendix F provides us with normal percentiles, but many calculators and statistics computer packages provide these as well.

15

FINDING NORMAL PERCENTILES BY HAND (CONT.)

- Table Z is the *standard Normal* table. We have to convert our data to *z*-scores before using the table.
- Figure 6.5 shows us how to find the area to the left when we have a *z*-score of 1.80:

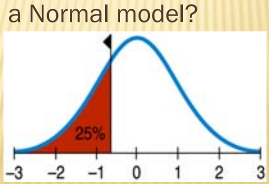


From *Stats Modeling the World* by Bock, Velleman, & De Veaux, 2010, p. 117.

16

FROM PERCENTILES TO SCORES: *Z* IN REVERSE

- Sometimes we start with areas and need to find the corresponding *z*-score or even the original data value.
- Example: What *z*-score represents the first quartile in a Normal model?



17

FROM PERCENTILES TO SCORES: *Z* IN REVERSE (CONT.)

- Look in Table Z for an area of 0.2500.
- The exact area is not there, but 0.2514 is pretty close.

- This figure is associated with $z = -0.67$, so the first quartile is 0.67 standard deviations below the mean.

18