Chapter 22

INFERENCES ABOUT MEANS

SAMPLING DISTRIBUTION FOR MEANS

- Recall, the Central Limit Theorem told us the sampling distribution for means

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

- What if we don't know σ ?

STANDARD ERROR

- We can approximate the standard deviation with the standard error:

$$SD(\bar{y}) = \frac{\sigma}{\sqrt{n}} \quad \approx \quad SE(\bar{y}) = \frac{s}{\sqrt{n}}$$

where s is the sample standard deviation

- For small sample sizes, this may not conform to the standard normal distribution so we instead use the Student's t-distribution

STUDENT'S T-DISTRIBUTION

- The t-distribution is different for different sample sizes
- It has the same general symmetric bell-shape as a normal curve, but reflects greater variability (wider)
- Mean t = 0
- Standard deviation varies with sample size, but is greater than 1
- As n (sample size) gets larger, the t-distribution gets closer to the standard normal distribution

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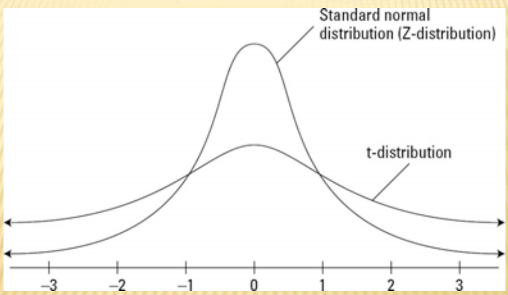

Retrieved from http://jpkc.njmu.edu.cn/course/tongjixue/file/jxzy/ybzzSD/images/fig14-2_0.jpg, March 29, 2010.

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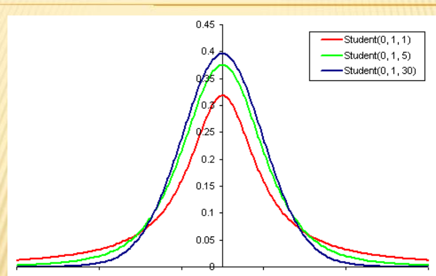

Retrieved from http://www.vosesoftware.com/ModelRiskHelp/images/13/image20.gif, March 29, 2010.

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DEGREES OF FREEDOM

- The number of degrees of freedom (df) for a single data set is the number of sample values that can vary after certain restrictions have been imposed on all data values
- For this application, df = n - 1

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SAMPLING DISTRIBUTION FOR MEANS

When the conditions are met, the standardized sample mean

$$t = \frac{\bar{y} - \mu}{SE(\bar{y})}$$

follows a Student's *t*-model with *n* – 1 degrees of freedom.

We estimate the standard error with

$$SE(\bar{y}) = \frac{s}{\sqrt{n}}$$

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ASSUMPTIONS AND CONDITIONS

- Independence assumption
  - Randomization
  - 10% condition
- Normal population assumption
  - Nearly normal condition: The data come from a distribution that is unimodal and symmetric.
    - Check this condition by making a histogram or Normal probability plot.

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ONE-SAMPLE *T*-INTERVAL

- When the conditions are met, we are ready to find the confidence interval for the population mean, μ.
- The confidence interval is

$$\bar{y} \pm t^*_{n-1} \times SE(\bar{y})$$

where the standard error of the mean is

$$SE(\bar{y}) = \frac{s}{\sqrt{n}}$$

- The critical value t^*_{n-1} depends on the particular confidence level, *C*, that you specify and on the number of degrees of freedom, *n* – 1, which we get from the sample size.

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ONE-SAMPLE *T*-TEST FOR THE MEAN

- The conditions for the one-sample *t*-test for the mean are the same as for the one-sample *t*-interval.
- We test the hypothesis H_0: $\mu = \mu_0$ using the statistic

$$t_{n-1} = \frac{\bar{y} - \mu_0}{SE(\bar{y})}$$

- The standard error of the sample mean is

$$SE(\bar{y}) = \frac{s}{\sqrt{n}}$$

- When the conditions are met and the null hypothesis is true, this statistic follows a Student's *t* model with *n* – 1 *df*. We use that model to obtain a P-value.

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TESTING HYPOTHESES ABOUT A MEAN

- The three possible choices for hypotheses are:
  1. H_0: $\mu = \mu_0$, H_A: $\mu \neq \mu_0$
  2. H_0: $\mu = \mu_0$, H_A: $\mu < \mu_0$
  3. H_0: $\mu = \mu_0$, H_A: $\mu > \mu_0$
- μ_0 is the <u>null value</u>

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