Chapter 18

CONFIDENCE INTERVALS FOR PROPORTIONS

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STANDARD ERROR

- Estimates the theoretical standard deviation of the sampling distribution for sample proportions based on a single sample:

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

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A CONFIDENCE INTERVAL

- By the 68-95-99.7% Rule, we know
  - about 68% of all samples will have $\hat{p}$ within 1 *SE* of *p*
    - So we are 68% sure *p* lies within one SE of $\hat{p}$
  - about 95% of all samples will have $\hat{p}$ within 2 *SE*s of *p*
    - So we are 95% sure *p* lies within two SEs of $\hat{p}$
  - about 99.7% of all samples will have $\hat{p}$ within 3 *SE*s of *p*
    - So we are about 99.7% sure *p lies within three SEs of* $\hat{p}$
- These are <u>confidence intervals</u>

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CONFIDENCE INTERVALS

- An interval of values that is fairly certain to contain the true value of the population parameter of interest
- The degree of confidence reflects the frequency of times that the confidence interval actually does contain the population parameter, assuming that the estimation process is repeated a large number of times

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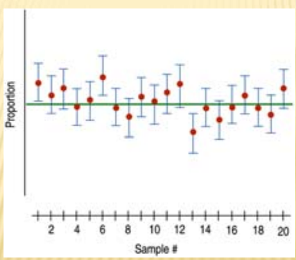
VISUALIZING CONFIDENCE INTERVALS



From *Stats Modeling the World* by Bock, Velleman, & De Veaux, 2010, p. 443.

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MARGIN OF ERROR: CERTAINTY VS. PRECISION

- We can claim, with 95% confidence, that the interval $\hat{p} \pm 2SE(\hat{p})$ contains the true population proportion.
  - The extent of the interval on either side of $\hat{p}$ is called the margin of error (*ME*).
- In general, confidence intervals have the form *estimate ± ME*.
- The more confident we want to be, the larger our *ME* needs to be.

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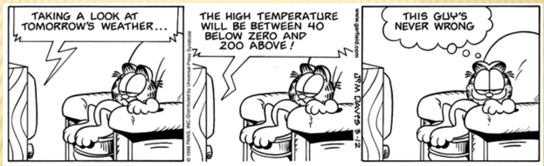
MARGIN OF ERROR



From *Stats Modeling the World* by Bock, Velleman, & De Veaux, 2015, p. 478.

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CRITICAL VALUES

- The '2' in $\hat{p} \pm 2SE(\hat{p})$ (our 95% confidence interval) came from the 68-95-99.7% Rule.
- Using a table or technology, we find that a more exact value for our 95% confidence interval is 1.96 instead of 2.
  - We call 1.96 the critical value and denote it z^*.
- For any confidence level, we can find the corresponding critical value.

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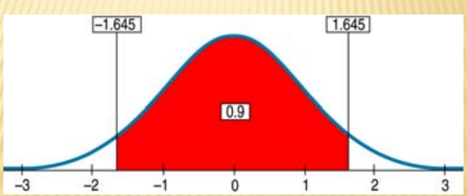
CRITICAL VALUES (CONT.)

- Example: For a 90% confidence interval, the critical value is 1.645:



From *Stats Modeling the World* by Bock, Velleman, & De Veaux, 2015, p. 479.

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ONE-PROPORTION Z-INTERVAL

- The confidence interval for the population proportion *p* is

$$\hat{p} \pm z^* \times SE(\hat{p})$$

where

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

- The critical value, z^*, depends on the particular confidence level that you specify.

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INTERPRETING THE INTERVAL

Don't Misstate What the Interval Means:

- Don't suggest that the parameter varies.
- Don't claim that other samples will agree with yours.
- Don't be certain about the parameter.
- Don't forget: It's the parameter (not the statistic).
- Don't claim to know too much.
- Do take responsibility (for the uncertainty).

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CHOOSING YOUR SAMPLE SIZE

- In general, the sample size needed to produce a confidence interval with a given margin of error at a given confidence level is:

$$n = \frac{\left(z^*\right)^2 \hat{p}\hat{q}}{ME^2}$$

where z^* is the critical value for your confidence level.

- To be safe, round up the sample size you obtain.

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